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NOTICES

Chairman Elect

At a Council Meeting held on June 5th, Mr. A. Ogilvie, C.B.E., Fellow, was elected Chairman for the year 1923-1924. Mr. Ogilvie will assume office on October 1st.

Donations

The following additions to the Library have recently been acquired:—

A collection of negatives and lantern slides illustrating the early experiments of the late Sir Hiram Maxim—presented by Lady Maxim.

Three books of cuttings containing a large number of letters from, and other information relating to the work of, the late Laurence Hargrave, collected by the late Mr. H. Crosland Taylor—presented by Mrs. Crosland Taylor.

Associate Fellowship Examination

The second examination for Associate Fellowship will take place in the Library, on Monday, September 24th (Part I.), and Tuesday, September 25th (Part II.). Intending candidates should forward entry forms (accompanied by the prescribed fee) on or before Monday, August 27th, stating the subjects in which they desire to be examined.

Journal

It is regretted that owing to the large amount of extra work thrown upon the staff by the International Air Congress, which was held from June 25th-30th, the appearance of this number of the Journal has been delayed.

W. LOCKWOOD MARSH, *Secretary.*

XUM

PROCEEDINGS

TENTH MEETING, 58TH SESSION

A meeting of the Society was held at the Royal Society of Arts on Thursday, March 1st, 1923, Professor L. Bairstow, C.B.E., F.R.S., in the chair.

The CHAIRMAN, in calling upon Major F. M. Green to read his paper on "Air Travel, with Special Reference to the Helicopter," said that many of those present would be able to agree with the very critical review of the problem given by the lecturer, and it would be an advantage perhaps to the progress of aviation if the Society gave a technical opinion as to the value of the helicopter as a means of air travel.

Major GREEN, before reading his paper, apologised for its being short, but pointed out that it was meant to be a basis for discussion. If there were points which he had misrepresented he would be very glad to receive advice, and he was certain that a full discussion would benefit the progress of aeronautics.

AIR TRAVEL WITH SPECIAL REFERENCE TO THE HELICOPTER.

BY MAJOR F. M. GREEN, FELLOW.

A striking development of the present century has been the use of air travel. A new method of going from place to place has come into use which is quicker than any former means. It is only since the war that definite air services have been available to the general public, and in consequence it is only right to believe that its development is in an early and crude state. At the present moment, although there are a number of types of aircraft engaged in this work, they are all similar in principle. It is suggested from time to time that we are possibly working on wrong lines and that air travel can be carried out more effectively using machines of quite other types. The suggested alternatives to the present aeroplane are airships and another form of heavier-than-air craft generally called the "helicopter" or direct lift machine. It is not proposed in this paper to discuss the relative merits of the aeroplane and the airship, but to state some of the outstanding disadvantages of the types of aeroplane now in use and to consider whether the "helicopter" offers a hopeful solution.

Objects of Air Travel

The usual object of air travel is to go from place to place quicker than is possible by any other means. Apart altogether from its greater speed, the aeroplane saves time by travelling more directly from point to point, and by avoiding changing for sea journeys. Nevertheless, the aeroplane must be able to maintain a high speed through the air in order to make effective progress against adverse winds. Experience has shown that the slowest cruising speed that is practicable for most routes exceeds 80 miles an hour. If the route chosen is likely to be fairly free from high winds and if the existing methods of travel are very slow, a slower speed might be useful. In a general way, however, the speed must be high to make it worth while using a method of travel which for some time to come must remain expensive.

Safety

Let us agree that it is useless to run an air service with machines with a cruising speed below 80 miles an hour; we may next consider what are the desirable attributes of a machine suitable for air travel. I think that safety must be the most important attribute of any aircraft. Later on we shall consider the question of cost of running, but it cannot be doubted, even if we neglect other considerations, that an aeroplane which is not reasonably free from chances of accident is not likely to be economical in service.

Reliability

Assuming that we have an aircraft of sufficient speed and safety, the next requirement is that, when a start is made, the passenger shall be reasonably certain of reaching his destination. The value of speed disappears very rapidly if, in more than a very small percentage of cases, the journey has to be finished by other means, or if a long stoppage has to be made en route. It has been said that air travel is either the quickest or the slowest means of travel—the quickest when everything goes right and the slowest if anything goes wrong.

Closely allied to the certainty of arriving at one's destination is regularity, for a transport service is of little use unless it can be trusted to operate at regular intervals.

There must also be a certain degree of comfort, or perhaps it is better to say a minimum degree of discomfort, otherwise the number of passengers likely to use air travel must be limited.

It is scarcely possible to give in figures the actual values of the requirements mentioned, though it is clear that if we are to make air travel into an ordinary commercial undertaking we must use aircraft in which the chance of accident to a passenger is no greater than the risk by train, motor-car or boat, while the time for the journey must be less than the best achieved by any combination of these means of travel. Regularity, reliability and comfort must approach the standard of the train.

The advantages of increased speed will encourage a proportion of the ordinary travelling public to pay a higher fare which will almost certainly be needed for a number of years. How much extra they will pay must depend upon the extent to which the service possesses the qualities mentioned. In my opinion, it is unwise to concentrate entirely on reducing the cost of running the service; rather we should endeavour to encourage passenger traffic by increasing the safety, speed, regularity and reliability of the service.

If we agree with the foregoing views our attention may be chiefly directed to safety combined with a certain minimum speed, and it is chiefly from these two aspects that the present-day aeroplane and its possible developments will be considered. I know of no example of a helicopter or direct lift machine which has achieved the smallest measure of success as a means of air travel, so that the comparison between the advantages of the two types is difficult. An attempt will be made to see what chance the helicopter has of becoming a rival to the aeroplane, and in order to do this we must touch on the first principles of mechanical flight. I do not propose to go deeply into the matter in a scientific way, as the experimental work on the helicopter is limited.

First Principles

Heavier-than-air machines obtain their support from the air by giving to it a downward velocity. The momentum developed downwards of the air per second is a direct measure of the lift obtained. This applies to both aeroplane and helicopter. With the aeroplane the air is driven downwards by means of planes of suitable shape which are drawn through the air by the reaction of an airscrew. It may be said that the name airscrew is misleading, for an airscrew develops

its tractive force not by screwing its way through the air (as the name seems to imply), but by projecting air backwards. Here again the reaction of the airscrew can be measured in terms of the momentum in the air displaced per second.

In the case of the helicopter the machine is sustained by projecting air downwards by means of wings or airscrews revolving in a plane which is more or less horizontal. The mechanism for obtaining support in the two cases is thus similar, but there is the wide difference that whereas the aeroplane uses the velocity of its lifting surfaces to carry it directly towards its destination, the helicopter planes or propeller blades, whichever you like to call them, have their main motion round a centre which is either fixed relative to the air or requires additional energy to move it. Diagram 1 shows this graphically—in the one case the planes of the aeroplane move from A to B in a straight line; in another they travel along a tortuous path. The energy required is roughly proportional to the length of the line joining A and B, hence it can be seen that the helicopter is at a very serious disadvantage.

Power Used for Support

The planes of a type commonly used in present-day aircraft have a ratio of lift to drag of about 15 to 1 when flying at their usual cruising speed. At 80 miles an hour in air of normal density we find that each effective horse-power spent on the planes alone corresponds to resistance of 4.7 lbs. and thus enables a weight of 71 lbs. to be supported. Taking a propeller efficiency of 75 per cent.—a figure usually obtained in practice—the weight supported per engine brake horse-power is 53 lbs. If we increase the speed of the aeroplane and use smaller planes, keeping the lift-drag ratio constant, the horse-power expended in flight will increase in direct proportion to the increased speed. This does not mean that as far as the planes are concerned more energy or more fuel will be used in travelling a given distance, but it does mean that the engine horse-power must be greater.

In the case of the helicopter it is generally understood that it is not possible to construct a direct lift airscrew that can lift nearly as much as 53 lbs. per horse-power, and if it were possible to do so the weight of the revolving planes themselves would be likely to exceed the weight lifted. The reason for this is simple—the support from the lifting screw is obtained by virtue of the downward momentum of the air in the slipstream. The air has a certain kinetic energy imparted to it which is entirely lost as far as the flying machine is concerned. The kinetic energy is proportional to the square of the velocity of the air in the slipstream, and it may be shown that to achieve a lift of 53 lbs./horse-power implies that the downward velocity must nowhere exceed 21ft./sec., and would in practice need to be less than this. This means that an airscrew to lift 1,000 lbs. must have a diameter of between 30ft. and 40ft., which is scarcely practicable.

This calculation is quite elementary, and is given below.

Lift of helicopter = mass of air dealt with per second × downward velocity imparted

$$= Mv.$$

$$\text{Horse-power} = \text{kinetic energy lost per sec.}/550 = Mv^2/2 \times 550 \text{ minimum.}$$

$$\therefore \text{Lift/H.P.} = Mv/Mv^2/1100 = 1100/v$$

M is probably not greater than $\rho v A$ where $A =$ " disc area " of helicopter.

$$\therefore \text{Lift}/A = Mv/A = \rho v^2, \quad \rho = .00237.$$

If W = weight of helicopter,

$$v = 1100/W/\text{H.P.}$$

$$\therefore W/A = .0023 \times (1100)^2/W/\text{H.P.})^2$$

$$\therefore \text{for } W = 1000$$

$$d = .67\ W/\text{H.P.} \text{ where } d = \text{diameter of helicopter in feet.}$$

and for a lift of 53 lbs./H.P. $d = 35$ feet minimum.

PATH OF AEROPLANE
A
B
A
B
PATH OF ELEMENT OF HELICOPTER

Power Required for Flight

In order to bring the size of lifting screw to a reasonable figure we must increase the downward velocity of the air and consequently the lift in lbs./horse-power must be less. It seems, therefore, that the power required for keeping the aircraft in flight is likely to be much greater in the helicopter than in the aeroplane (see Fig. 2). The resistance of the body, landing gear, and controlling organs of the helicopter is not likely to be less than that of the aeroplane, and we may assume it to be the same. There is, however, an additional resistance due to the framework, driving mechanism and so forth of the revolving propeller blades which is likely to be greater than the resistance of the structure and wiring of the conventional aeroplane.

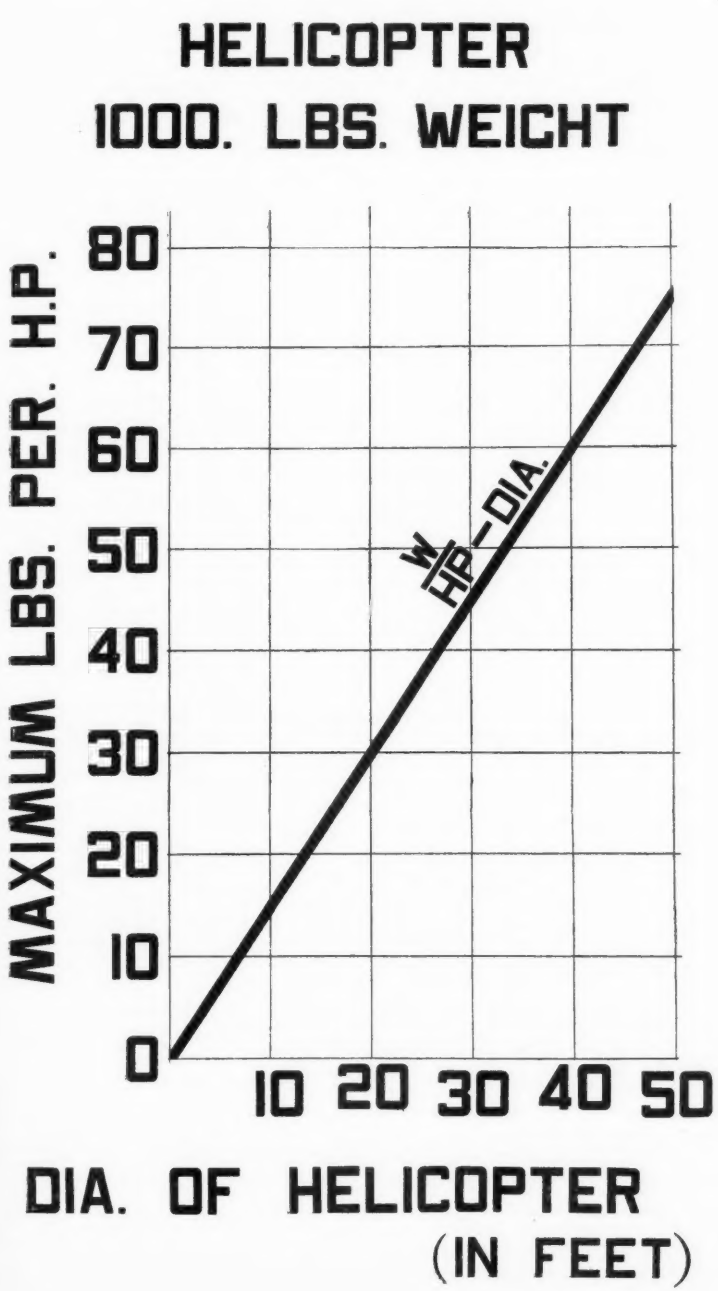
The effectiveness of the lifting screw will be seriously disturbed by any forward motion imparted to the helicopter, for it will mean that at one point of its rotation the propeller blade will have an additional velocity imparted to it by the forward motion of the whole machine, while at another this speed will be reduced by an equal amount. It has been proposed by many inventors to make the angle of the blades vary throughout each revolution. This, however, involves additional mechanism, and the construction of it is scarcely likely to be simple or light. In any case there will be an additional resistance to be added to the whole resistance due to the movement of the revolving propellers through the air.

From the foregoing reasons I am convinced that the power expended in flight in a helicopter, flying at speeds found to be useful for aeroplanes, will be very much greater than for the aeroplane, and I believe that, apart from all other disadvantages, this fact alone will render the machine quite impossible for passenger carrying, at any rate until engines of much less weight per horse-power and materials with much greater specific strength are available. With present-day materials it is my opinion that it is extremely unlikely that it will be possible to make a direct lift machine carrying any useful load which will be able to fly as fast as 80 miles per hour, which is the slowest cruising speed that makes flying worth while in most cases.

Safe Landing

If the conclusions of the last paragraph are correct, it seems that as a means of air travel the helicopter has little or no future, and the fact that it may be possible to rise and land in confined spaces is of little value if the ability to fly from place to place at a reasonable speed is absent. In itself, the advantage of a vertical rise and fall is unlikely to be as great as might be supposed. The difficulty of effecting a landing of a direct lift machine in a wind is certain to be great. In the ordinary aeroplane the presence of a wind helps rather than hinders matters both in getting off and alighting. The case of the helicopter is worse than that of the airship, where considerable skill is necessary to effect safe landings in a high wind.

The idea of being able to lower your flying machine to the ground by the careful working of the throttle is a pleasing one, and at first sight it seems vastly to be preferred to the aeroplane method of approaching the ground with a forward speed of 50, 60 or more miles an hour. The fact remains, however, that aeroplane pilots do not find much difficulty in landing at these speeds so long as they are not forced to alight in unexpected places. The usual reason for forced landings is, of course, the failure of the motive power, and it will be interesting to see how the helicopter compares with the aeroplane in this emergency. The aeroplane method is to glide down to the ground at a speed somewhat above the stalling speed and to use the kinetic energy of the machine to level up and fly parallel to the ground for the last few moments before alighting. So long as there is room, this method presents no difficulties. In the case of engine failure in the helicopter the situation is rather different. Neglecting all difficulties of


HELICOPTER
1000. LBS. WEIGHT
MAXIMUM LBS. PER. H.P.
80
70
60
50
40
30
20
10
0
W/HP-DIA.
10 20 30 40 50
DIA. OF HELICOPTER
(IN FEET)

stability and assuming that the helicopter will keep more or less on an even keel, the whole machine will descend vertically in relation to the air, and its fall will be checked chiefly by the resistance of the supporting propellers. When the engine stops the propellers will either, after stopping, be driven round in the opposite direction or they may have their pitch reversed and travel in the same direction. The terminal velocity of the whole machine unfortunately must be high as the propellers do not offer very much more resistance when spinning than when stopped.

A reference to the experiments on the resistance of an airscrew on an aeroplane will show that the increase of resistance is only of the order of 10 per cent. when it is spinning at the velocity which gives the maximum resistance, as against when it is stopped. It is possible that a specially-designed screw would have a somewhat higher resistance than this, but there seems no reason to suppose that much can be gained. The helicopter, therefore, if unchecked, will strike the ground much faster than is convenient or can be readily dealt with by even an elaborate form of shock-absorbing gear. In addition, the whole machine will be moving relative to the ground at or nearly the velocity of the wind.

There is only one means of checking the fall of the machine known to me other than by the use of parachutes. The method is similar to that used by the aeroplane pilot—the supporting propellers will be revolving at a fair speed and will have in consequence a kinetic energy in virtue of their rotation. If at the last moment the propeller blades are reversed in angle, then their kinetic energy can be used to check the fall much in the same way as is done on the aeroplane. Whether this can be considered a feasible method or not is a matter of opinion, but it is the only method which I can suggest in case of engine failure on the direct lift machine. In any case, the results with forced landings are likely to be serious, for even if the rate of fall is checked there is sure to be some forward velocity due to the wind.

Landing in Fog

Landing in fog when it is not possible to see the aerodrome, the helicopter machine certainly does appear to possess its chief advantage. On foggy days there is usually little or no wind, and it ought to be possible to allow the helicopter to descend vertically at a sufficiently slow rate to avoid serious shock, even if the ground is completely invisible. We have as yet devised no safe method of landing aeroplanes in fogs, and although there is no reason to suppose we shall not eventually manage this, it is likely to be a difficult matter and will require a great deal of organisation. As is known, the usual method of landing an aeroplane is to arrange its flight path so that it approaches the ground at as small an angle as possible. The aeroplane is flown at a speed exceeding the stalling speed, and either the throttle is opened slightly near the ground or else the angle of the planes is increased and the kinetic energy of the whole machine is used to supply the power.

It is generally assumed that if the angle of an aeroplane is increased so that it flies above its stalling angle, disaster is almost certain unless there is room to dive the machine and to regain sufficient speed to fly below the stalling angle. On aeroplanes as usually made, this is to a large extent true. The controls are insufficient to enable the pilot to manage the aeroplane when flying at an angle above the stalling angle. It is as well to mention that by the stalling angle is meant that angle at which the planes exercise their maximum lift coefficient, and when any increase in angle will not increase and may decrease the total lift coefficient. There appears no reason why it should not be possible to fly at angles much above the stalling angle. This subject has been the matter of research during the past year, and it has been found possible to fly an aeroplane at angles of incidence vastly greater than the stalling angle. It seems not unlikely that in the future it will be a safe manœuvre to glide an aeroplane at an angle

of incidence of as much as 45 degrees, and still maintain control. The drag of the planes will be very large, and the path of the aeroplane will be about 45 degrees to the horizontal; consequently the fuselage of the aeroplane will remain nearly horizontal. In the case of a forced landing, or a landing in the fog, it might even be possible to bring an aeroplane down to the ground at an angle of 45 degrees, in which case the forward speed will be so much reduced that the aeroplane will run a very small distance after landing; also owing to the steep angle the errors of judging distance on the ground will be very much smaller. If the engine has not broken down it will probably be possible to straighten up the machine by means of the engine and effect a more or less normal landing. If the engine has broken down, or if it is impossible to see the ground, then it will be necessary to provide a landing gear designed to absorb a much bigger shock than is now customary. In any case it is almost certain that the vertical velocity on landing will be considerably lower than will be possible with a helicopter machine with the engine stopped.

Stability

Another aspect of safety is the question of stability. It is, of course, possible to make an ordinary aeroplane stable in flight, and so long as the pilot is not undertaking any severe manœuvre, such as is sometimes made necessary by engine failure, the risk of accident from loss of stability is unimportant. The stability of a direct lift machine has, I believe, never been worked out numerically. It will certainly be a matter of considerable complication, and it may be anticipated that the structural design to meet the various forces that may occur on the direct lift machine due to sudden manœuvre will vastly increase the difficulty of designing what in any case must be a somewhat complicated mechanism.

Structural Safety

From the point of view of structural safety the helicopter is likely to present grave difficulties. The aeroplane depends for its support upon a system of planes which, except for small movements of the control surfaces, are fixed. In the helicopter the equivalent of the plane structure is dependent upon a number of bearings and working joints which will certainly increase the difficulty of making a safe structure that is reasonably light.

Conclusions

The brief discussion of the problems contained in this paper is meant to represent the argument which occurred to me when considering whether or not it was worth attempting the design of a direct lift machine. It is perfectly true that there are many considerations which would prevent a private constructor starting on such an undertaking which would not, and should not, influence the minds of those directing the official policy of a country in matters of aeronautics. The private constructor must be influenced by financial considerations to a greater extent than would be the Research Department of the Ministry. At the same time, there is only a limited amount of money that can be spent on research and experiment in aeronautics. We have been informed at the last two Air Conferences that experiments on full scale have been, and are being, carried out by the Air Ministry, but we have not been supplied with any details either of the way in which the experiments are being made or of the results obtained, with the sole exception that a year ago we were informed that free flight had taken place. Recently there have been rumours of large prizes offered by the Government for any machine capable of doing certain performances, which include hovering. I do not know the precise arguments which led the Air Ministry to undertake work of this description, but for the reasons given in the former part of the paper it seems to be improbable that any useful result will be obtained unless we can make vast improvements in the technique of the production of power and the making of light structures; such advances would also improve the design and performance of the ordinary aeroplane.

It will no doubt be argued that there are peculiar advantages that might be gained in war from a machine capable of hovering, but if this is so it is suggested in all seriousness that a balloon or an airship is a far more promising method of obtaining the required result. It is likely to cost less and to be safer. The object of air travel is to get from place to place, and it seems highly unlikely that the helicopter type of machine will ever afford a useful means of doing this.

Discussion

Mr. J. Lawrence Hodgson, B.Sc., Assoc.M.Inst.C.E., opening the discussion, said that in the early years of the war he had carried out some very careful experiments on model propellers (including tests on propellers whose plane was inclined to the direction of motion) in order to determine whether it was practicable to build a helicopter. The results of some of those tests were given in a paper which he had read before the Institution of Automobile Engineers, entitled "Tests on Model Propellers," in 1916.

The broad conclusions he had come to were entirely in agreement with those that Major Green had advanced, although his tests had shown that it was quite possible, even with our present development of engines and structures, to build a helicopter which would get off the ground and fly.

A point on which he was in disagreement with Major Green was his statement that when a helicopter was flying horizontally it required more horse-power than when it was hovering. Actually it required much more horse-power to get a helicopter vertically off the ground than it did to fly it at 40 m.p.h. This was because, when a helicopter was travelling forward, the propellers were continually engaging undisturbed air. This is illustrated by the tests summarised in Table I. These tests were taken in water with a flat-bladed propeller. The size of the blades was 0.75in. × 0.4in. (the long axis being set radially). Their thickness was 0.01in. The distance between the centres of the two plates forming the blades was 2.25in. The blades were set at an angle of 15 degrees to the plane of motion. The propeller was immersed in water which was flowing parallel to the plane of the propeller at the velocities tabulated. In each series of tests the thrust was kept constant by diminishing the revolutions and the torque as the velocity of the water past the propeller was increased.

It will be seen that the thrust per horse-power is very much increased when the fluid in which the propeller is working is moving parallel to the plane of rotation of the propeller. There is a similar, though smaller, increase when the fluid moves at a small angle to the plane of the propeller. When, however, this angle is increased to more than about 20 degrees, the tests show that the thrust per horse-power ceases to increase as the relative velocity between the propeller and the fluid in which it works is increased.

The tests quoted are two out of a considerable number which were taken in 1915 on flat-bladed propellers, whose plane of rotation was inclined at various angles to the direction of motion of the fluid in which the propellers worked.

Table II. gives some of the actual observations, and Table I. the faired-up results.

TABLE I.

Vel. of water at right angles to the propeller axis in feet per sec.	Thrust per horse-power in lbs. Thrust 0.08 lbs.	Thrust 0.16 lbs.
0	190	135
1	250	164
2	315	200
3	395	245
4	480	283
5	570	330

TABLE II.

Vel. of water at right angles to the propeller axis in feet/sec.	Thrust 0.08. Torque (in. lbs.).	Revs. per min.	Thrust 0.16 Torque (in. lbs.).	Revs. per min.
0	0.042	648	0.0843	915
2.26	0.0309	480	0.0642	738
3·75	0.0281	397	0.0575	619
4.90	0.0253	285	0.0574	558

The fact that the thrust per horse-power of the propellers *does* increase as the helicopter moves forward is one that should be clearly recognised by the opponents as well as by the advocates of the helicopter. If Major Green had added to his diagram comparing the flight paths of the wing of an aeroplane and the blade of a helicopter screw, the path of the blade of the propeller actuating the aeroplane, the comparison would not have been so greatly to the disadvantage of the helicopter.

If one imagined a helicopter which had horizontal motion before it got off the ground (see Fig. 20 of the paper* before the I.A.E.), such a machine could get off the ground by running along and gradually climbing, and would require very little more horse-power than the equivalent aeroplane. Similarly, if the engine broke down and it was still possible to maintain the stability of the machine, it could glide down as the aeroplane does. His tests (which he believed were the first ever taken on propellers having a negative speed of advance) further showed that if the engine-power of a helicopter was not all contained in one unit—supposing it had four engines, three of which broke down—it would be possible, by maintaining the screws rotating at one quarter of their full horse-power, to come down vertically at by no means dangerous velocity. Yet, in spite of all this, when one took into consideration the extraordinary complexity of the helicopter, the difficulty of balancing, the difficulty of landing accurately without damaging the propellers, or of landing at all in a confined space when any wind was blowing, the conclusion he had definitely come to was that—since the aeroplane had been developed, it was foolish, when the amount of money available for research was limited, to try to develop the helicopter, which would necessarily be a much more cumbersome and complex mechanism for achieving flight. This was stated as follows in his paper before the Institution of Automobile Engineers :—

> " The final conclusion come to was that a helicopter which would fly successfully could be built, if desired. Such a machine appeared, however, to have many disadvantages as compared with the aeroplane, such as the great weight of the gearing necessary (at least 1½lbs. per horse-power), the unsuitability of any of the ordinary forms of power transmission for use on a lightly-built and easily distorted framework, the very great liability to mechanical breakdown under gunfire, the enormous gyroscopic forces induced in the propellers if the inclination of the machine were suddenly changed, the serious danger of straining the propellers if the landing were at all faulty, and the almost imperative necessity for the use of gyroscopic control with its attendant complications.
>
> " The most practical solution of the helicopter problem seemed to be that shown in Fig. 24, where two aeroplanes circle round an observation car, which, by means of gearing, is prevented from rotating.
>
> " This solution of the helicopter problem, while eliminating most of the difficulties above enumerated, did not seem to be sufficiently promising to be worthy of further development."

Continuing, Mr. Hodgson said he believed certain Governments (he would not say which) were experimenting on the lines essentially of two aeroplanes tied

* Reprinted in Appendix I. of the present Paper.

together at the wing tips and circling round a stationary observation car for the purpose of maintaining a definite position in order to take observations. He thought expenditure of public monies on such experiments was very much to be regretted.

Mr. A. FAGE said that he considered Major Green had made out a very good case against the helicopter, and he believed a case which could be strongly supported at the present time. In connection with the descent of a helicopter with the engine cut out, he wished to refer to some experiments recently made by Messrs. Lock and Bateman at the N.P.L. The results of these experiments had not yet been fully analysed, so that he spoke with some reserve. The experiments seemed to show that even with the blades at a small negative angle, the resistance of a helicopter was unlikely to be greater than that of a flat plate of the same disc area. This meant that if for a helicopter of 1,000 lbs. the speed of descent were limited to 16ft. per second, the diameter would have to exceed 60ft.

Major LOW recalled his experience of helicopter design as far back as 1912, when he had to work out in detail a general design put forward to his firm (Messrs. Vickers) by a well-known aeronaut.

The engine was given and the dimensions of the airscrews, and the problem was to design the framework drive, etc. When the details were worked out on orthodox lines with the minimum factor of safety that could be accepted, the structural and power plant weights far exceeded the most optimistic estimates of the lift to be expected.

The instruction was then changed to that of getting out a light enough framework regardless of such trifles as factors of safety, but this having been done, it was reported that in the estimation of the design staff the whole apparatus would collapse as soon as the engine was opened out. This restrained the original designer from going ahead on his own responsibility, and the matter was finally dropped. At the time aeroplanes were being designed with a reasonable performance as a matter of routine.

Major Green's paper showed that the very same difficulties were still present, and that although improvements in glider and power plant weight coefficients had made very considerable advances, yet relatively the helicopter was hopelessly handicapped, and always would be.

He would like to draw the conclusion implied in the paper and in the remarks of a previous speaker. If they moved the principal weights outward and made them move with the moving blades or wings of the helicopter, then in descent at a vertical speed comparable with that of an aeroplane, and with the wings supposed adjustable to the best gliding angle, at the last moment before landing the incidence could be altered to that of maximum lift by stages, the energy being supplied by the rotating masses, and a landing made with no vertical velocity in a manner exactly analogous to an aeroplane. But the design would approximate in this extreme case to two normal aeroplanes attached at the wing tips and flying round each other in narrow circles. He did not require to explain there how hopelessly inferior such a combination would be. Major Green had also omitted to consider the effect of horizontal movement in increasing the effective lift of a helicopter, but even with this apparent advantage there would be the countervailing disadvantage of greatly increased stresses, requiring great increase in weight, and moreover if a helicopter had to run along the ground to attain flying speed, farewell to the machine that was to rise vertically from a restricted space. Major Green had done a great service if his paper moderated ill-informed enthusiasm, and dispelled the fascination exercised by the idea of hovering over the lay mind.

Mr. F. HANDLEY PAGE said he had not heard the paper read, but when he had read the title "Air Travel, with Special Reference to the Helicopter," he

had thought what a marvellous opportunity this was for the proposed new Civil Aviation Company. Those who might have had doubts as to what were the objects of the Company, or on what its funds would be expended, would at last find some object sufficiently large on which the capital could go. (Laughter.) He did not know whether anyone had taken part in the discussion who was a very great advocate of the helicopter, because he could quite imagine someone pointing out that twenty years ago Lord Kelvin had said that flying was impossible with an ordinary aeroplane, and someone might say that twenty years from now the helicopter would be a possibility. He personally thought it was somewhat dogmatic, with all due deference to the lecturer, to say that the helicopter, although perhaps not so efficient as the aeroplane, would not be of some use twenty years hence.

There was one very interesting semi-helicopter which he had seen in Madrid a short while back, and he did not know whether members of the Society had seen a description of it. This consisted of an ordinary aeroplane fuselage at the centre of gravity of which was fixed an inclined vertical shaft, the angle of inclination being a few degrees back from the vertical. On this inclined shaft was what appeared to be a four-bladed propeller, between 15ft. and 20ft. in diameter, the blades consisting of fabric-covered planes of approximately 1ft. chord, the cross section of each being ordinary aeroplane section. These planes were set at zero angle of incidence to the plane of rotation. Instead, however, of this propeller being driven round positively by the engine, it was free to rotate. The fuselage was fitted with an ordinary engine propeller, and when the machine commenced to run along the ground to take off like an ordinary aeroplane, the blades of the horizontal propeller started to rotate, and owing to the forward component of the vertical reaction on the blades, the blades rotated in a way which seemed at first contrary to what one would expect, namely, the leading edge of the aerofoil section moved forward. Owing to the increased speed obtained by the velocity of rotation, a higher lift was obtained than if the planes were stationary. To equalise the lift on the two sides, a very ingenious device was incorporated by the inventor. The blades were pivoted at their interior end, approximately 2ft. from the axis of rotation, and the interior end was held in position by rubber shock-absorber cord. In consequence of this arrangement on the side in which the velocity of rotation and translation were the same, the blade was allowed an upward movement, thereby diminishing the effective angle of attack. The reverse effect took place on the other side, so that the angle of attack was increased and the lift correspondingly increased. By this means the lifts on the two sides were equalised.

This machine, although somewhat complicated, had actually flown, and seemed to alight very slowly. Whether, however, such a device was really good and worth the complication involved is open to question.

Colonel HECKSTALL SMITH said he was afraid the lecture would have a most depressing effect on the Press, because one thing the Press really loved was a helicopter, and he would like the lecturer to let the members of the Society know, if possible, what results had been distributed among designers as the result of the almost encouraging expressions of opinion made by the Air Ministry at the Air Conference before last, held, he believed, more than a year ago at the Guildhall. We had been told by the Under-Secretary of State for Air that "Substantial progress had been made towards the successful solution of one of *the chief problems* of aeronautics—that of vertical flight by means of a helicopter." The emphasis on "one of the chief problems" is mine, but the importance of such an expression in view of this lecture is noteworthy. Further we were told that "efforts are proceeding and public money is being spent" and our hopes were raised, so much so that the papers were full of it, but from that day to this he did not think anything further had been heard. It would be well if the Society could obtain some information as to what had been done, so that designers who

might have been encouraged to take up what the Air Ministry had suggested was such a good line of thought might know whether it was still a good line of thought. At the same time, he hardly considered that, after Major Green's lecture, anyone connected with designing would be very much encouraged to go on in that direction.

Major WIMPERIS said that it would seem that the more successful a helicopter design proved to be, the greater would be its similarity to two ordinary tractor machines fastened together and rotating round a common centre. If this were so, since we have plenty of good tractor machines now, we could, he supposed, by combining a pair of them so as to rotate about a common centre, have the best form of helicopter already. Major Green had called his paper "Air Travel, with Special Reference to the Helicopter," but the helicopter's special reference was not to air travel, but to "hovering." No one would think of fastening two tractor machines so as to face in opposite directions as a means of air travel, though they might certainly do so if "hovering" were the object.

The CHAIRMAN, referring to the fact that not much had been heard about the Air Ministry experiments on helicopters, said the members of the Society would like to know exactly what scientific advice the Air Ministry had taken before putting them in hand.

Major GREEN, in reply to the discussion, expressed his disappointment that there was no one present who had sufficient faith in the helicopter to speak on its behalf. Mr. Hodgson's remarks were very interesting to him, and he was sorry that he had not come across Mr. Hodgson's paper, because it seemed to him that had he done so, he would not have read his own paper at all. Mr. Hodgson must have covered the ground much more adequately than he himself, and also had some actual experiments on which to base his figures, whereas he himself had not made experiments but had had to depend on first principles and such information as he could collect. With regard to the helicopter taking less power when moving forward, he could quite imagine that, because they were approaching nearer and nearer to the aeroplane. Anyhow, he would be surprised if the helicopter, when actually moving forward, did better than the figures given in Diagram 2, because in that case he had taken no account of interference in the ordinary way, and he had assumed that all the air moved by the propeller blades was moved downwards with uniform velocity. It was very unlikely, unless they approached so near to the aeroplane that the apparatus would cease to be a direct lift machine at all. He believed, in any case, that Mr. Hodgson and himself were in complete agreement, and he was pleased to have Mr. Hodgson's confirmation of the results which he had arrived at by other means.

With regard to flattening out, of course, if they put the weight further and further outside, they did approach Major Wimperis's two aeroplanes tied together. That was a helicopter of sorts, but as a means of air travel it did not seem to possess any great advantages. It seemed to him that, if they could find a skilful pilot, he could, by banking his machine sufficiently, go round in such small circles in an ordinary aeroplane that he could almost hover. With two machines tied together they neutralised the centrifugal forces, which enabled them to go round in small circles, but he thought it was unlikely to be a success as a form of travel.

He had not heard Mr. Fage's remarks, but he gathered he had said that the fact that the planes were revolving did not help very much. The suggestion that perhaps they could design a special sort of airscrew did not seem to be justified.

Major Low had given some very sound advice. He believed he could explain to Major Low why the helicopter had such a fascination for the inventing type of people, and possibly for the official mind. When quite a small boy he had read a book called "The Clipper of the Clouds," which described an airship which was a sort of boat with a lot of lifting screws. That had made a big impression on

his mind, and he supposed on those of other people, and he thought that was really why so much attention was given to this direct-lift business.

With regard to Mr. Handley Page, he did not quite follow the advantages of the machine he had sketched, but if it had achieved flight it had achieved something. Mr. Handley Page had suggested that he (Major Green) had been rather dogmatic, but he believed Mr. Handley Page would withdraw his remarks if he read the paper carefully. He (Major Green) had not said the helicopter could not fly, but merely that, as a means of air travel, for the reasons given in the paper, there did not appear to be much hope for it, and therefore there were other things it would pay better to develop than to spend our limited resources on the helicopter.

He had anticipated such a question as that asked by Colonel Heckstall Smith. Therefore, he had written officially to the Air Ministry asking whether he might be supplied with any information with regard to direct-lift machines which they had advertised as having achieved free flight rather more than a year ago. He had received the usual reply—that nothing further could be added to the information already given. That suggested two reasons. One was that it was a frightfully important experiment that was being carried out, and the other reason he left his hearers to imagine. When he had mentioned to one man that it was stated at the Air Conference that the helicopter had achieved free flight, the reply was " So has my umbrella." A friend of his had visited a cinema on the day previous to the meeting, and had seen a picture of a machine which carried two gentlemen. According to the pictures the machine had achieved a free flight of two minutes; it had got off the ground to the extent of two or three feet, and had moved in an aimless way, and the pilot apparently landed without smashing himself. He did not dispute that free flight had been achieved, but in his paper he was dealing with the helicopter as a means of air travel.

He joined with Professor Bairstow in asking on what scientific advice the experiments had been made on direct-lift machines, but he was afraid that the reply would be that it was not in the interests of aviation that the information should be divulged. He daresay that would be right.

A hearty vote of thanks to the lecturer concluded the proceedings.

Dr. WATTS (*communicated*): Major Green has expressed very clearly what I think is the considered opinion of most technical men in this country who have seriously considered the problem of helicopters.

I have often illustrated the fundamental difference between the helicopter and the aeroplane by likening them to two men who one night came out of Trafalgar Square tube station both feeling very cold. They most correctly decided that in the circumstances the only way to get warm was to run. But one—the helicopter man—ran round and round Trafalgar Square; the other man ran straight home. They both got warm, but one arrived home and the other arrived nowhere.

There is a point which I think the lecturer has not sufficiently emphasised, that is the performance of a helicopter at any respectable altitude, and I think I might usefully quote from my contribution to the discussion on M. Damblanc's paper before this Society.

There referring to M. Damblanc's proposals, I am reported thus:—

" The lift he obtained worked out at 13½lbs. per horse-power. This was not an exaggerated figure. He (the speaker) would expect a higher figure. He thought 20lbs. gross lift per horse-power might easily be obtained. If not, the helicopter would be of no particular value. The weight of the helicopter proposed by M. Damblanc was 2,640 lbs.; the lift at ground level was about 3,000 lbs. The ceiling could therefore be little more than 5,000 feet. At the ceiling all the thrust would be required merely to support the machine and the translational velocity would therefore be zero. Under these conditions its powers of manœuvring would be almost negligible, and he put it to the meeting that an aircraft with only a ceiling of 5,000 feet, with no trans-

lational velocity at that height and with little power of manœuvring, had no serious military value. It would be particularly vulnerable to anti-aircraft fire and to attack from quite ordinary aeroplanes."

I feel quite safe in saying that if a helicopter ever becomes possible it will only be possible for very low altitude work.

For the reasons which Major Green has given and those which I expressed in the discussion alluded to, I very much deplore the expenditure which has been and possibly is still being incurred by our Air Ministry in research on helicopters, and I should again like to quote from a contribution I made to a discussion at the Second Air Conference, 1922. I said :—

"The research which is now being carried out on helicopters is a research which, in the interests of economy, might well be stopped, without involving any serious loss of Great Britain's position in aviation. I am sure that in making that statement I shall carry with me most of the technical men in this room.

"Generally speaking, in deciding its policy of research, the Government have been guided by its Advisory Committee in the past or the Aeronautical Research Committee in the present, or by its own technical advisers in the Air Ministry itself. I do not think that the research on helicopters which we are told is now proceeding has ever been suggested or asked for either by the past Advisory Committee or by the present Aeronautical Research Committee. I would go further and say that the research was started against the advice of the Air Ministry's own technical advisers. I speak with some knowledge of the facts since I was myself one of those technical advisers at the time when this research was initiated.

"I do not wish to enter into technical details, but there seems to be a large amount of misunderstanding concerning the difficulties to be overcome in the evolution of a helicopter of any commercial or military value. Both at this Conference and in the newspapers of other countries we have heard it almost triumphantly announced that a helicopter has succeeded in rising a few feet under its own power. This result seems in the mind of many to justify the thousands of pounds which have been spent. The Director of Research has suggested to this Conference that it is an encouraging result.

"The helicopter is nothing more or less than a propeller exerting a thrust vertically instead of horizontally, and I confidently assert that for many years we have had sufficient knowledge of propellers to have predicted this result without the expenditure of money which has taken and is still taking place. The technical men of this country have not asked for research on helicopters, not because they doubted whether such a machine could rise vertically from the ground, but because they could not see of what use it would be, or how it could be controlled and rendered stable when it had done so. I do not, as I have said, wish to go into technicalities, but I would suggest that the Secretary of State for Air should seek the advice of his Aeronautical Research Committee or any other external authority on the question whether research on helicopters is justified in the present stringent financial circumstances."

Captain DE HAVILLAND (*communicated*): A reasoned estimate of the capabilities of the helicopter has been badly wanted, and is now supplied by Major Green's paper.

It is true that there are many experiments which could be justifiably undertaken with the aid of public funds which no private concern could entertain in the present condition of the industry.

The kind of experiment referred to is that in which there are many difficult practical problems to be solved—generally at considerable cost—but in which there

is a definite gain in view; a gain (of which some estimate may be made) over existing methods of doing the same work.

It is this preliminary case for the helicopter which never seems to have been made out, and any attempt to do so leads to conclusions similar to those reached by Major Green, who, in order to make discussion of the problem possible, has certainly not exaggerated the difficulties.

It is difficult to avoid the conclusion that the whole experiment is a waste of money.

TESTS ON MODEL PROPELLERS

BY J. LAWRENCE HODGSON, B.SC., ASSOC.M.INST.C.E.

Reprinted from "Proceedings of the Institution of Automobile Engineers."

Appendix I.

The investigation summarised in the two preceding papers formed part of a rapidly carried out research which was made at the commencement of the war in order to determine whether a machine on the helicopter principle could be built, and whether, when built, it was likely to prove of any special value for belligerent purposes.

The method of flight upon which the idea of the helicopter is based was probably invented by Nature far earlier than the "flapping wing" method. Microscopic rotifers of very ancient ancestry, whose waving *cilia* enable them to hover, and to move freely in three dimensions, may be discovered in stagnant pools.

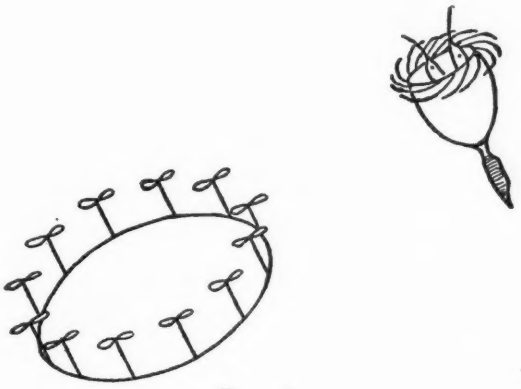


FIG. 16.

Fig. 16 shows one of these creatures, and also the diagram of an equivalent arrangement in which propellers are employed.

Leonardo da Vinci was probably the first who seriously considered the helicopter principle as a possible basis for achieving flight. The actual machines which he attempted to construct were, however, based upon the "flapping wing" principle.

Many years ago Jules Verne's book, "The Clipper of the Clouds," gave a popular exposition of the possibilities of the helicopter.

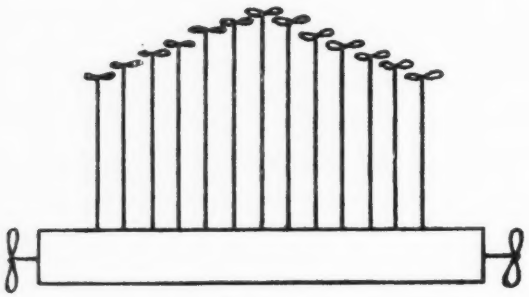


FIG. 17.

The machine which was therein described, Fig. 17, was faulty in conception, in that it should have travelled broadside on, instead of lengthways, in order to avoid interference effects between the lifting propellers, while the forward motion could have been more easily obtained by tilting the whole machine in the plane of motion. Such a re-modelled " Clipper of the Clouds " is shown in Fig. 18.

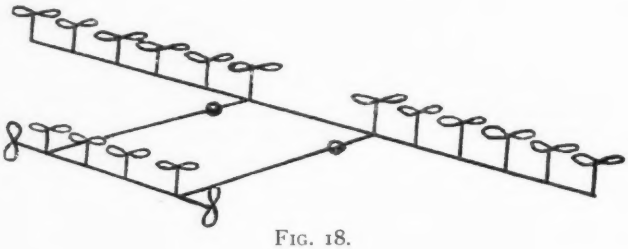


FIG. 18.

If the torque and the gyroscopic effects are to be balanced, and the machine is to be able to tilt itself, the least number of propellers that can be employed is 4.

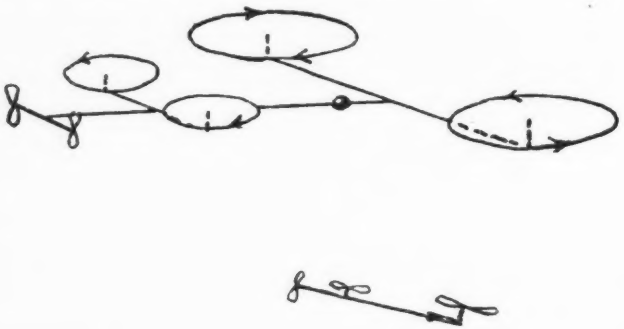


FIG. 19.

A four-screw helicopter is shown diagrammatically in Fig. 19. In this figure there are two additional screws for steering, and for turning the machine when it has no velocity of translation.

It was assumed that such a machine, if built, would have a central power plant of, say, three or four independent engines, and that there would be some means of transmitting to the propellers all the power which was developed by the power plant in the event of the failure of one or more of the engines. It was therefore of interest to determine the speed of vertical falling when the power delivered to the propellers was too small to enable the machine to rise vertically. For this reason the values of the revolutions and the torques at various *negative* speeds of advance, given in Figs. 4 and 5, were determined. Calculations based upon the results there given show that quite a large proportion of the engine

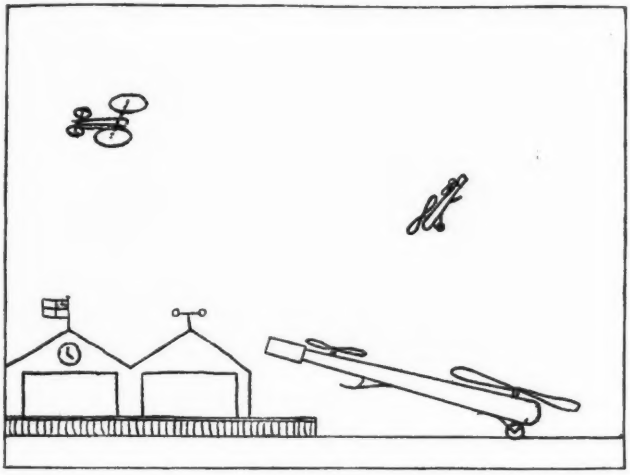


FIG. 20.

power necessary to hover may be lost without impairing the power of the machine to descend vertically at by no means dangerous velocities.

A number of tests taken with the apparatus shown in Fig. 15, with propellers inclined to the direction of motion, showed that it would be possible to glide down with safety even if the engine power failed altogether; provided, of course, that the balancing arrangements were such that the stability of the machine was not thereby impaired, and that the machine was fitted with the necessary landing wheels.

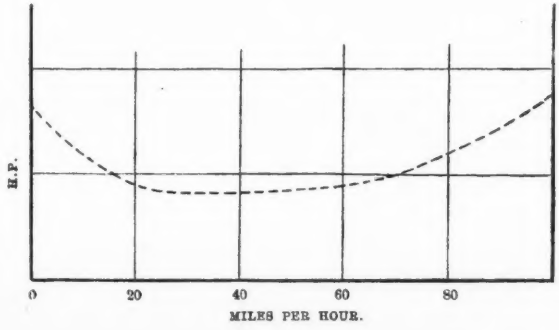


FIG. 21.

The same series of tests upon inclined propellers also showed that it would be possible to construct helicopters of very much smaller size than the unwieldy hovering machine first considered; provided that these smaller helicopters ran along the ground so as to obtain an initial velocity before rising. Such machines are indicated in Fig. 20.

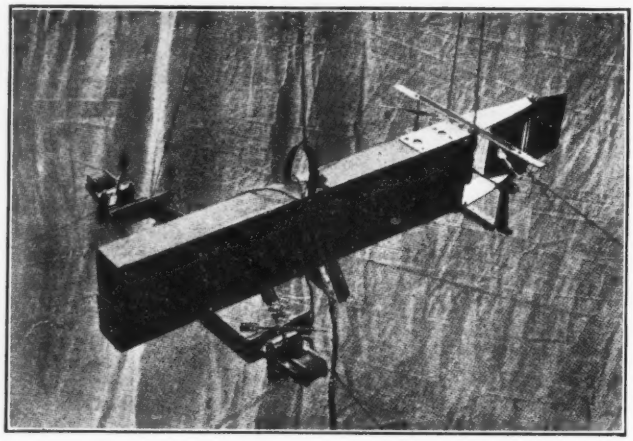


FIG. 22.

Another interesting point which was brought out by the tests on inclined propellers was that the horse-power required to enable a helicopter to hover was sufficient to propel it horizontally at over 80 miles per hour, and about double that required to propel it horizontally at 40 miles per hour (see Fig. 21). This is due

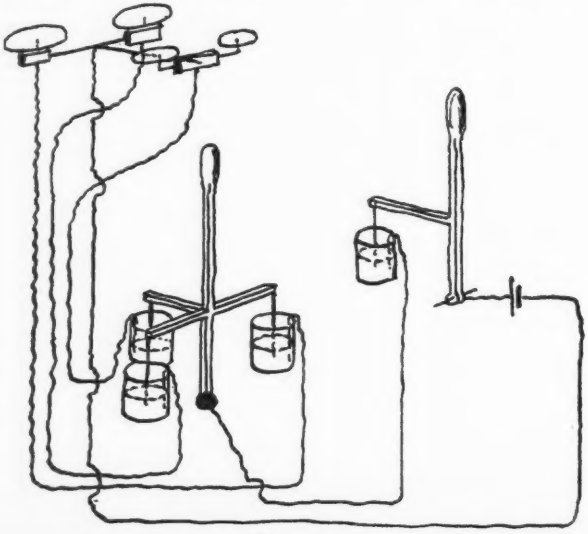


FIG. 23.

to the greater volume of air "engaged" by the propellers when the machine travels forwards.

In order to test the difficulty of balancing a helicopter when in flight by means of a hand control apparatus, the electrically-driven model shown in Fig. 22, Plate XV., was made. This model "flew" in water—hence its rather bizarre proportions—and was controlled by means of liquid resistances, as indicated in Fig. 23. It was found to be possible to control a model in this way very easily.

In spite of this, however, it was felt that if an actual machine were built, some form of gyroscopic control would be necessary.

The final conclusion come to was that a helicopter which would fly successfully could be built, if desired. Such a machine appeared, however, to have many disadvantages as compared with the aeroplane—such as the great weight of the gearing necessary (at least 1½lbs. per h.p.), the unsuitability of any of the ordinary forms of power transmission for use on a lightly built and easily distorted framework, the very great liability to mechanical breakdown under gunfire, the enormous gyroscopic forces induced in the propellers if the inclination of the machine were suddenly changed, the serious danger of straining the propellers if the landing were at all faulty, and the almost imperative necessity for the use of gyroscopic control with its attendant complications.

CONSOLIDATED WOOD

A NEW MATERIAL FOR VARIOUS INDUSTRIES

BY W. R. TURNBULL, M.E., F.R.AE.S.

When engaged on war work as Chief Inspector for Fredk. Sage & Co. at Peterboro', England, in the spring of 1918, I observed that when Honduras mahogany was compressed across the grain, and parallel to the annular rings of the wood, a "first" or "natural" elastic limit was reached, after which the deflections *increased* more rapidly than the load, as has been commonly observed in all testing of all materials.

If, however, we continue increasing the load, crushing does not occur as one would expect, but after a period, as the load is increased the deflections *decreased*, and this decrease was more and more accentuated up to the point at which the load crushed the sample under test, which point was *many times* (15 to 17 times or more) the "natural" elastic limit of the wood.

This is illustrated in Curve "A," Fig. 1, which shows the general characteristics of three methods of compressing wood with typical results, viz. :—(*a*) Wood compressed across the grain and parallel to the annular rings, (*b*) across the grain at right angles to the annular rings, and (*c*) "end compression"—the load being parallel to the grain on the end of the fibres.

I also found that if the load was thus carried, in compression parallel to the annular rings, far beyond the elastic limit, but was stopped before the crushing point was reached, and the wood was removed from the press, this wood, which I term "consolidated wood," will "recover," and in two or three days assume a permanent "set," after which any change in its dimensions will be extremely slow (see Note on "Recovery").

After ten weeks the samples of Honduras mahogany, thus "consolidated," were subjected to various tests, and it was found that this "consolidated wood" *was much stronger* in every way than the natural wood from which it was made, and moreover the consolidated wood had *little tendency to shrink* when exposed to a high temperature (see Note on "Effects of Heat").

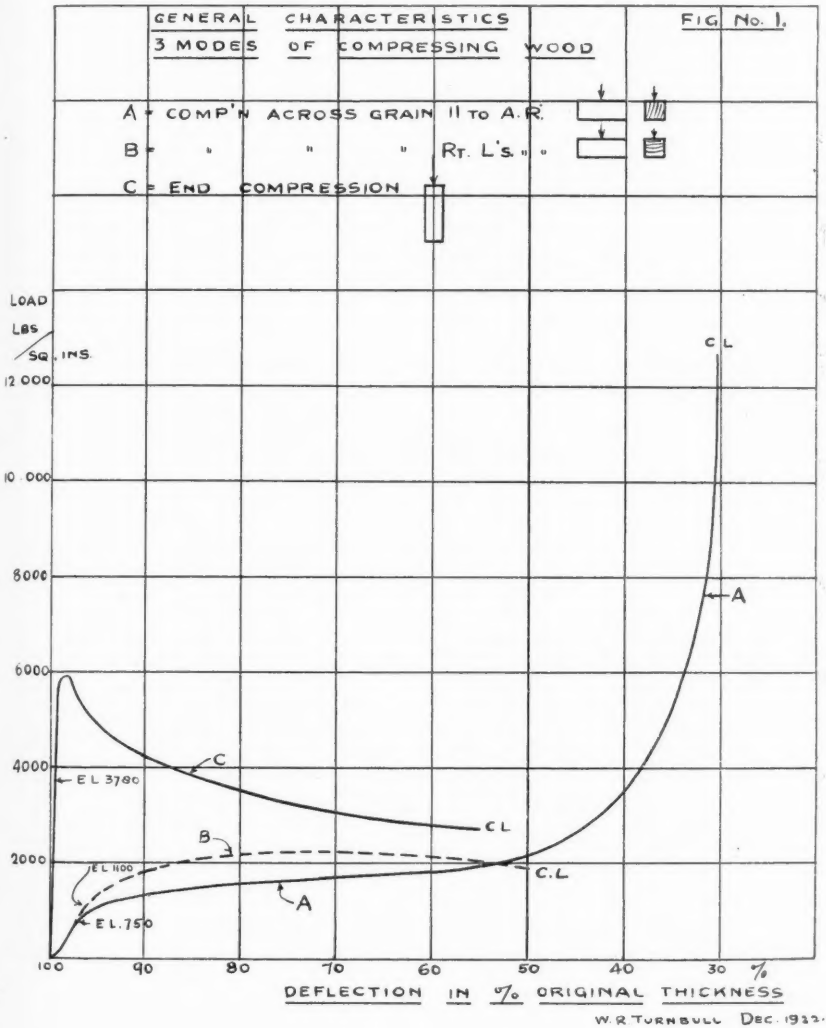
I have since found that these properties of Honduras mahogany are not peculiar to this wood alone, but that all woods, probably, when consolidated have the same properties to a greater or less degree; but it is possible that deciduous ("hard") woods have the properties to a greater degree than the "soft" woods, although this is a matter for further investigation, as my tests have been limited to six woods only.

The reason for the increase of strength in wood treated in this way is rather obscure, but in some tests, in which samples of seasoned birch were consolidated (in a mould) to 15,000 lbs. per square inch, it was noted that a gum was squeezed from the ends of the samples, and it is therefore believed that this gum acts as a natural cement, which holds the fibres in closer contact than that in which they naturally grow, and thus a stronger and denser wood is formed.

When wood is consolidated the density is increased, but in general the proportionate increase in density is considerably less than the proportionate increase in strength. Compare Figs. 2, 3 and 4.

I have been unable to carry out a very extended research in this interesting subject, but I here present the results as far as my work has gone.

For convenience of reference and comparison the results are best shown by the curves of percentage increase plotted against the amount of previous consolidation in lbs. per square inch in Figs. 2, 3 and 4.



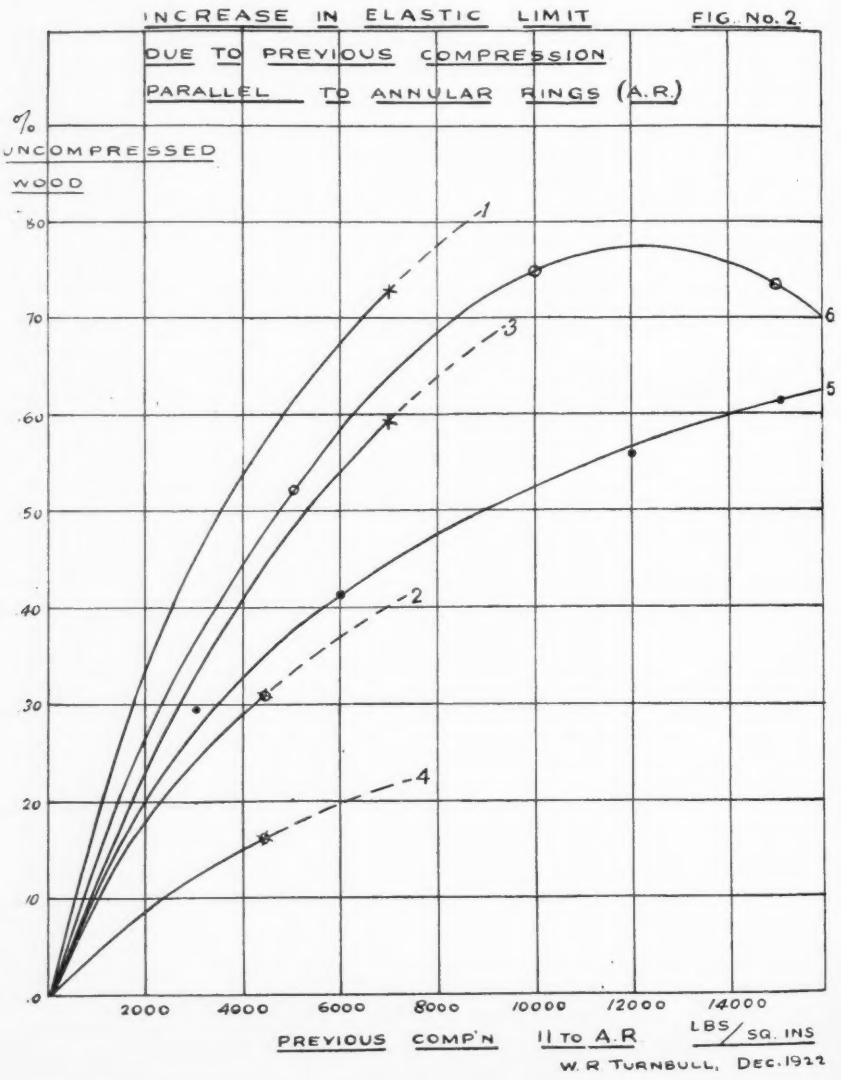
In these we have the results for four woods that have been consolidated and later tested as follows :—

(1) Compression across the grain at right angles to the annular rings, (2) compression on the ends of the fibre "end compression," and (3) bending.

The number of the curves are tabulated as to wood and method of test as follows; and the same numbers apply to the same woods, and method of testing, in each of the Figs. Nos. 2, 3 and 4.

Curves No. 1.—Honduras mahogany, tested in compression across the grain and at right angles to the annular rings.
" " 2.—American white oak, tested as above.
" " 3.—Honduras mahogany, tested in end compression.
" " 4.—American white oak, tested in end compression.
" " 5.—Yellow birch, tested in bending.
" " 6.—American hickory, tested in bending.

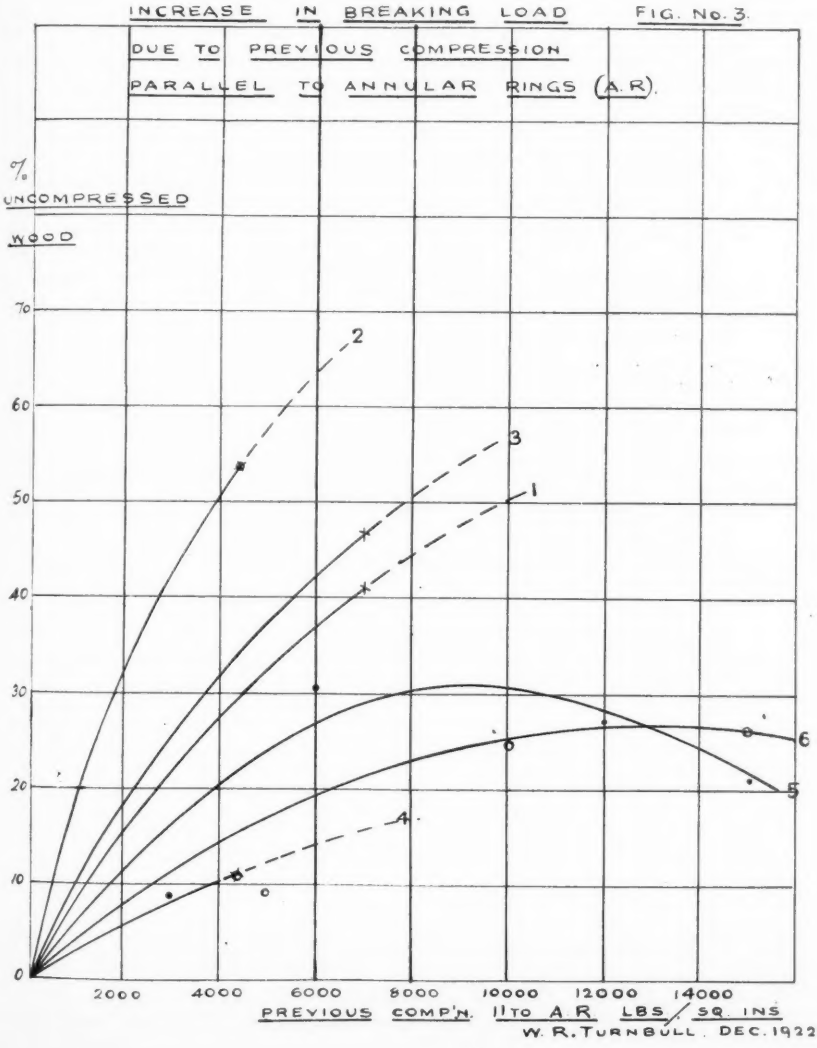
(When the samples were first consolidated, the sides were not confined for Nos. 1 to 4, but in the case of 5 and 6 the sides of the samples were confined by an iron " mould " to prevent side-bulging.)



Referring to Fig. 2, it will be noted that the elastic limit is increased by previous "consolidation" from 22 to over 80 per cent., depending on the wood and the manner of testing; and in the case of hickory, under bending test (Curve No. 6) a well-defined maximum occurs at about $77\frac{1}{2}$ per cent. increase and 12,000 lbs./sq. ins., and in comparing this with Fig. No. 4 we note that a maximum in density (lbs./cubic ft.) also occurs at about this point of consolidation.

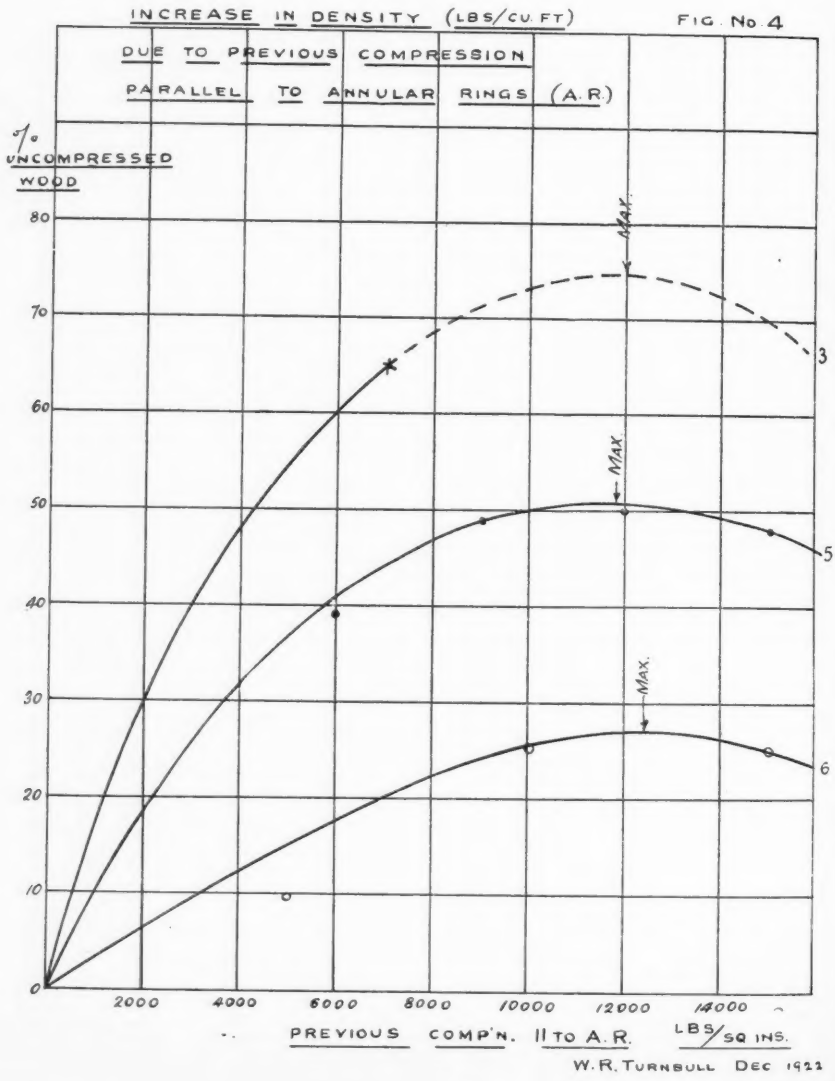
In fact, it seems probable that no useful purpose will be served by compressing the (deciduous?) woods beyond this consolidating load of 6 tons per square inch.

Referring to Fig. 3, we note that while there is a very material increase in



the breaking load, it is not so marked as in the cases for elastic limit; again, a maximum seems indicated at about 12,000 (10,000 for birch, 14,000 for hickory, both broken in bending tests).

The data on density increase (Fig. No. 4) is not so complete as would be desirable, but maxima are indicated, in the three cases given, in the vicinity of a consolidation pressure of 12,000 lbs./sq. in.; comparing these Curves 3, 5 and 6 with those of Fig. No. 2 (and corresponding numbers), we see that density increase is not so great as strength increase.



For Honduras mahogany it is apparently about the same, but for the other two we have :—

Birch maximum increase in density = 51%. Do. elastic limit = 62+.
Hickory ,, ,, ,, = 27%. ,, ,, ,, = 77·5%.

Note on "Recovery."—In some samples of Honduras mahogany that had been consolidated (7,000 lbs./sq. in.) over *four years* previously, the recovery (towards original thickness) was only from 6 to 7½ per cent.

Note on "Effects of Heat."—Samples of Honduras mahogany that had been consolidated (7,000 lbs./sq. in.) were heated, after six months, to a temperature of 200 deg. F. for 1½ hours, and shrank about 2½ per cent.; they then stood for three years and had recovered 1½ per cent.; they were again heated (120deg. F. for 21 hours) and shrank about 1 per cent., so that the dimension changes, due to extreme temperatures (for wood), are of the order of 1 to 2 per cent. only.

It is obvious, from the general results here given, that we have to deal with a rather remarkable phenomenon when we compress woods across the grain and parallel to the annular rings, and it is a phenomenon that should have some very useful applications in the arts and industries.

Such a common wood as birch can be given the strength and density of such a rare wood as English boxwood by consolidating it with a load of 6 tons per square inch, and many uses should be found (where wearing qualities, strength and resistance to shrinkage by heat are of importance) for such an artificial wood, or similar ones, in many industries.

Consolidated wood should be of great use for aeroplane parts and air propeller parts, for farm implement parts, spokes of wheels, shuttles for mill work, etc.; in short, wherever a strong, dense wood is required for special purposes.

I regret placing before the Royal Aeronautical Society a paper that is somewhat "sketchy," but the stress of other work has prevented me going into this matter more fully, and the results here presented represent a good deal more work than is apparent on the surface.

It is to be hoped that some of our younger engineers who have a testing machine, and more time, available will be able to take up this interesting research and give us much more extended results on the subject.

TWO-DIMENSIONAL AEROFOIL THEORY

BY MURIEL GLAUERT, B.SC.

1 *Introduction*

The object of the present paper is to give some account of certain methods developed by Joukowsky, Mises and others for calculating the forces experienced by a body of aerofoil shape placed in a stream moving in a fixed direction with uniform speed. The span of the aerofoil is supposed infinite and the cross section uniform and the motion thus becomes two-dimensional.

The classical methods of calculating the forces on a body placed in an inviscid fluid moving with uniform velocity give no resultant force on the body. But if on this motion we super-impose a circulation round the body we obtain a resultant force at right angles to the direction of motion, *i.e.*, a lift, but no drag. The lift will depend on the magnitude of the circulation imposed. It is, however, difficult to see how such a circulation could be set up in an inviscid fluid.

Now although the viscosity in a fluid, such as air or water, is small, it appears that its effect on the motion is appreciable. Let us therefore examine the various ways in which viscosity might be expected to affect the motion.

The force of viscosity is proportional to the velocity gradient perpendicular to the stream at any point, and hence it may be expected to affect the flow appreciably only in the regions where the velocity is changing rapidly. This will be the case near the surface of the body, since at the surface the velocity is zero, and also in the region of the trailing edge where, owing to the rapid change of curvature, the velocity tends to become very great.

The effect of viscosity in the layer of fluid close to the surface of the aerofoil will be to produce a retarding force or drag. We here have an indication of the reason why in the non-viscous solution the drag force does not appear. Neglecting viscosity in this region may be considered equivalent to neglecting the drag of the aerofoil. As, however, the characteristic property of an aerofoil is its large lift compared with its drag, a solution assuming slip at the surface of the body might be expected to give a reasonable approximation to the motion.

There remains the high velocity in the region of the trailing edge. Joukowsky conceived the idea of a circulation round the body set up by the viscous forces to counteract excessive velocity at the trailing edge. The magnitude of this circulation is in general indeterminate, but Joukowsky, in his theory, uses aerofoils with a sharp trailing edge and it is then possible to find a value for the circulation such that the flow at this point is finite.

Thus we are led to the consideration of the flow round an aerofoil placed in an inviscid fluid moving with uniform velocity and having a circulation imposed on it.

Now the equations of motion for a circle placed in such a stream are known and by means of a fundamental theorem by Bieberbach we can use these equations to obtain the flow past a body of aerofoil shape.

2 *General Theory*

Bieberbach's theorem is as follows: Given a closed curve without double points, then there is one and only one transformation of the type

$$\zeta = z + b_1/z + b_2/z^2 + \ldots$$

where $\zeta = \xi + i\eta$, $z = x + iy$, and b_1, b_2, . . . are in general complex quantities of the type $a + ib$, such that the space external to the curve in the z plane is

transformed into the space external to a circle in the ζ plane in such a manner that the region at infinity remains unchanged. The transformation is analytic in the region external to the curve or circle, *i.e.*, there is a one to one correspondence between points in the z plane external to the curve and points in the ζ plane external to the circle. The position and magnitude of the circle will be determined by the values of b_1, b_2. . .

Conversely there is a conformal transformation of the type

$$z = \zeta + a_1/\zeta + a_2/\zeta^2 + \ldots \quad . \quad . \quad . \quad . \quad (1)$$

whereby a circle in the ζ plane is transformed into a contour similar to that of any given contour without double points.

Assuming the right hand side of (1) to contain $(n + 1)$ terms we have

$$z = \zeta + a_1/\zeta + a_2/\zeta^2 + \ldots a_n/\zeta^n \quad . \quad . \quad . \quad (2)$$

Then

$$dz/d\zeta = 1 - a_1/\zeta^2 - 2a_2/\zeta^3 - \ldots - na_n/\zeta^{n+1} \quad . \quad . \quad (3)$$
$$= (1 - v_1/\zeta)(1 - v_2/\zeta) \ldots (1 - v_{n+1}/\zeta^{n+1})$$

where $\Sigma v_1 = 0$, $\Sigma v_1 v_2 = a_1$, etc., and v_1, v_2, . . . v_{n+1} are the zeros of $dz/d\zeta$.

The transformation (1) is conformal, *i.e.*, elementary arcs retain their shape on transformation, their lengths being increased by a factor $|dz/d\zeta|$, while each element is turned through an angle arg $dz/d\zeta$.* This process, however, breaks down at points for which $dz/d\zeta$ is zero or infinite, and by taking one of the zeros of $dz/d\zeta$ on the circumference of the circle it is possible to obtain from the smooth contour of the circle a contour with a sharp edge at one point. As this may be taken as a first approximation to an aerofoil shape having the sharp point for its trailing edge, we consider a transformation of the type (1) applied to a circle having one of the zeros of $dz/d\zeta$ on the circumference and enclosing the remaining zeros. From the form of (3) we see that the origin has been taken at the C.G. of the zeros of $dz/d\zeta$.

Now consider a circle with centre M and radius a in the ζ plane (Fig. 1),

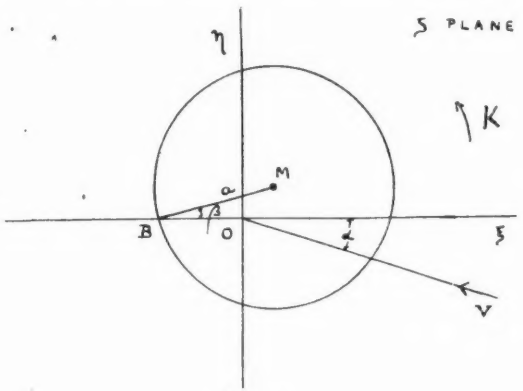


FIG. 1.

and let B be a point on the circumference at which $dz/d\zeta = 0$. Take as origin O of co-ordinates the centroid of the zeros of $dz/d\zeta$ and take BO for ξ axis. We shall suppose the cylinder having this circle for its cross section to be placed in a uniform stream V, the direction of the stream making an angle α with the negative axis of ξ, and there being a circulation round the cylinder equal to K.

* See Appendix I

For the motion let $w = \phi + i\psi$ where ϕ is the velocity potential and ψ the stream function. Then the magnitude of the velocity q at any point in the z plane is given by $|dw/dz|$.

But

$$dw/dz = (dw/d\zeta)/(dz/d\zeta)$$

and at the point B $dz/d\zeta = 0$. Hence, if the velocity at the trailing edge of the aerofoil in the z plane is to remain finite $dw/d\zeta$ must be equal to zero at B, *i.e.*, B is a point on the circumference of the circle at which the velocity vanishes.

To find this stagnation point consider the circle referred to axes $M\xi_1$, $M\eta_1$, through the centre, the direction of motion being that of the negative axis of ξ_1. Then from symmetry there will be two stagnation points A and B say, such that MA, MB make angles ϵ with the ξ_1 axis as shown in Fig. 2.

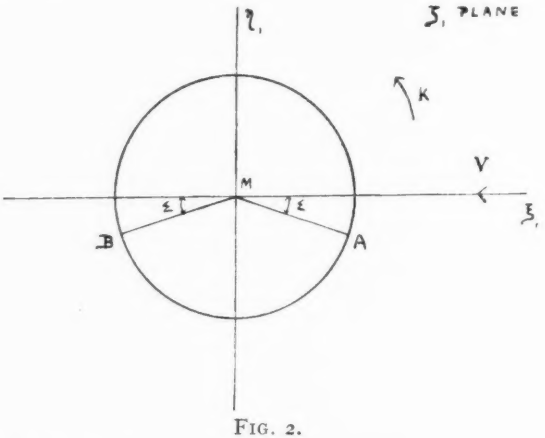


FIG. 2.

Then the motion of the fluid is given by*

$$w = \phi + i\psi = V\{\zeta_1 + a^2/\zeta_1\} + (iK/2\pi)\log\zeta_1 \quad . \quad . \quad (4)$$

where

$$\zeta_1 = \xi_1 + i\eta_1 = re^{i\theta}$$

Equating the real parts of (4) we have

$$\phi = V(r + a^2/r)\cos\theta - K\theta/2\pi$$

and if v be the component of velocity perpendicular to the radius vector at any point ζ_1, $v = -(1/r)\,\delta\phi/\delta\theta$.

At A and B $r = a$ and $v = 0$

whence

$$K = -4\pi aV\sin\theta$$
$$= 4\pi aV\sin\epsilon$$

Returning to Fig. 1 we have, if B is to be a stagnation point, $\epsilon = \alpha + \beta$ giving

$$K = 4\pi aV\sin(\alpha + \beta) \quad . \quad . \quad . \quad . \quad (5)$$

In order therefore that the velocity at the trailing edge should be finite, the circulation which must be imposed on the uniform stream V is determined by equation (5) and is seen to depend on α, the angle which the undisturbed stream makes with OB.

* See Lamb's "Hydrodynamics," Sections 68 and 69.

Lamb (*loc. cit.*) gives the following formula for the resultant pressure on the circular cylinder per unit length

$$L_1 = \rho VK$$

where ρ is the density, and this pressure is at right angles to the direction of the undisturbed stream.

Now since the motion is irrotational the circulation round any closed path enclosing the cylinder is constant. But the transformation (1) leaves the region at infinity unchanged and hence the circulation round the contour in the z plane is also K; the resultant pressure, or lift, is therefore given by

$$L = \rho VK \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$= 4\pi a\rho V^2 \sin(\alpha + \beta) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If M_1 is the moment of this lift about the centre M of the circle of transformation it is shown in Appendix I. that

$$M_1 = 2\pi c_1^2 \rho V^2 \sin 2(\alpha + \gamma) \quad . \quad . \quad . \quad . \quad (8)$$

where $a_1 = c_1^2 e^{2i\gamma}$

Thus equations (7) and (8) give the resultant force on the aerofoil and its moment about M.

Putting $\alpha + \beta = \alpha_1$ we have for (7)

$$L = 4\pi a\rho V^2 \sin \alpha_1$$

where α_1 is the angle of incidence measured from the no lift axis, *i.e.*, BM (Fig. 3).

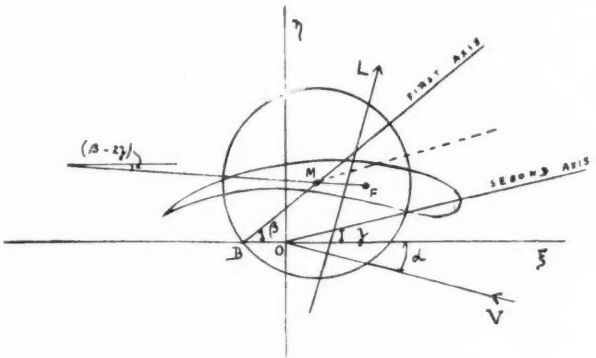


FIG. 3.

We shall call BM the first axis.

Again putting $\alpha + \gamma = \alpha_2$ we have for (8)

$$M_1 = 2\pi c_1^2 \rho V^2 \sin 2\alpha_2$$

and the moment of the lift about M vanishes when $\alpha_2 = 0$, *i.e.*, when $\alpha = -\gamma$.

We shall call a line through O making an angle γ with BO the second axis.

We notice

(1) That the lift depends only on the radius a of the circle and the position of the first axis.

(2) That the moment about M depends only on the constant c_1 and the direction of the second axis.

Two more interesting properties can be established.

(3) There is a point F about which the moment of the lift is independent of α. Vectorially

$$MF = (c_1^2/a)\, e^{i\,(2\gamma - \beta)} = (a_1/a)\, e^{-i\beta}$$

and the moment of L about this point $= -2\pi c_1^2 \rho V^2 \sin 2\,(\beta - \gamma)$.

(4) The lines of action of the lift as α varies envelop a parabola having BM as directrix and F as focus.

Proof of (3) and (4).

From M (Fig. 3) mark off a distance $MF = c_1^2/a$ in a direction making an angle $2\,(\beta - \gamma)$ with the first axis, the angle being measured in a clockwise direction. Then the second axis bisects the angle between MF and the first axis. (In the figure β is taken greater than 2γ.)

Let M be the moment of L about F. Then

$$\begin{aligned} M &= M_1 - L\,(c_1^2/a)\cos(\alpha - \beta + 2\gamma) \\ &= 2\pi c_1^2 \rho V^2 \{ \sin 2\,(\alpha + \gamma) - 2 \sin(\alpha + \beta)\cos(\alpha - \beta + 2\gamma) \} \\ &= -2\pi c_1^2 \rho V^2 \sin 2\,(\beta - \gamma) \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

i.e., M is independent of α.

Again, if $2h_0$ is the perpendicular from F on the first axis

$$h_0 = (c_1^2/2a) \sin 2\,(\beta - \gamma) \quad . \quad . \quad . \quad . \quad (10)$$

and if h is the perpendicular from F on the line of action of the lift

$$\begin{aligned} h = M/L &= -(c_1^2/2a) \sin 2\,(\beta - \gamma)/\sin(\alpha + \beta) \\ &= -h_0/\sin(\alpha + \beta) \quad . \quad . \quad . \quad . \quad . \quad (11) \end{aligned}$$

From the form of h we see that the foot of the perpendicular from F on the line of action of L lies on a line parallel to the first axis at a distance h_0 away from it, *i.e.*, lies on the tangent at the vertex of a parabola having F as focus and first axis as directrix (Fig. 4). It follows that the line of action of L is a tangent to this parabola.

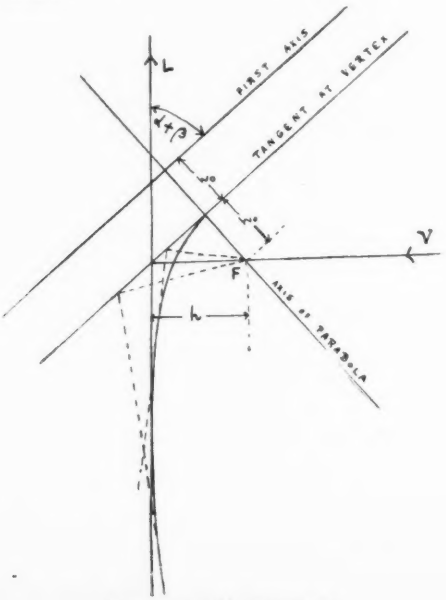


FIG. 4.—*Lift Parabola.*

An interesting case arises when $h_0 = 0$, *i.e.*, when $\beta = \gamma$ and the first and second axes are parallel. The lines of action of the lift now all pass through F and the aerofoil has a fixed centre of pressure.

From the above analysis we see that, given the transformation which will transform the circle into the given aerofoil, the characteristics of the aerofoil can then be found. The problem, however, of finding the necessary transformation is one of great difficulty and the method employed by Joukowsky and Mises has been to search for functions which would transform a circle into a contour approximating to an aerofoil shape.

3 *The Transformation* $z = \zeta + c^2/\zeta$.

Let us begin by considering a transformation containing the first two terms only of the series (1). We then have

$$z = \zeta + a_1/\zeta \qquad . \quad . \quad . \quad . \quad . \quad (12)$$

and

$$dz/d\zeta = 1 - a_1/\zeta^2$$

The zeros of $dz/d\zeta$ are at the points $\pm \sqrt{a_1}$ and the origin is at the centroid of these zeros. If now we choose the ξ axis to pass through the two zeros, a_1 will be real, and writing $a_1 = c^2$, the zeros of $dz/d\zeta$ are the points B, A, given by $\zeta = -c$ and $\zeta = +c$. According to the theory developed above one of the zeros, $\zeta = -c$ say, must lie on the boundary of the circle of transformation. Then B is the point on the circle which will transform into B', the trailing edge of the aerofoil (Fig. 5).

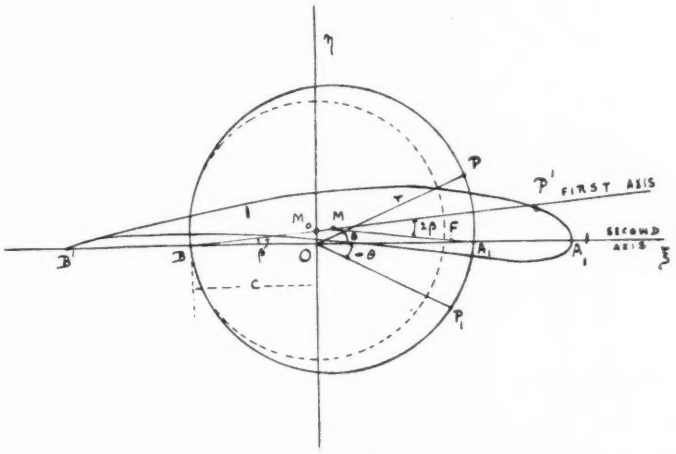


FIG. 5.—*Joukowsky Aerofoil.*

The other zero $\zeta = +c$ must lie within the circle. Let the ξ axis cut the circle again in A_1. As before M is the centre and a the radius of the circle and angle $MBO = \beta$. Then BM is the first axis.

Now $a_1 = c_1^2 e^{i\gamma} = c^2$ for the present case. Hence $c_1 = c$ and $\gamma = 0$ and the axis of ξ becomes the second axis. The line joining B and M will always cut the η axis in M_0 say, and the thickness and camber of the aerofoils obtained from the circle by means of the transformation (12) will depend on the quantities a and β. The focus is a point F in MA_1 such that $MF = c^2/a$.

The type of aerofoil obtained by the transformation (12) was first studied by Joukowsky and the aerofoils are known as Joukowsky aerofoils. Before proceeding to discuss the properties of these aerofoils and the method of constructing them we shall first consider the case when M lies on the η axis.

4 *The Circular Arc*

The contour obtained when M lies on the η axis does not belong to the class of aerofoils under consideration for now we have two zeros of $dz/d\zeta$ lying on the boundary of the circle, *i.e.* at A and B, instead of one (Fig. 6). Hence at the

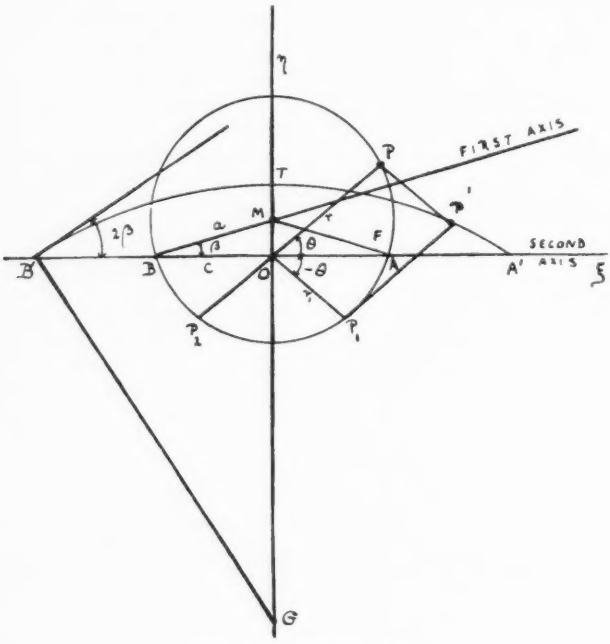


FIG. 6.—*Circular Arc.*

point A' on the contour corresponding to A on the circle the velocity will become infinite. As, however, the most suitable aerofoil shapes are obtained when M_0M and β are small, the limiting case when $M_0M = 0$ is of interest. The circle now transforms into a circular arc. For if $\zeta = re^{i\theta}$ be the point P on the circle, the corresponding point P' on the contour in the z plane is given by

$$z = \zeta + c^2/\zeta \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$x + iy = re^{i\theta} + (c^2/r)\, e^{-i\theta}$$

whence equating real and imaginary parts

$$x = \cos\theta\,(r + c^2/r)$$

$$y = \sin\theta\,(r - c^2/r)$$

Eliminating r we have

$$x^2 \sin^2\theta - y^2 \cos^2\theta = 4c^2 \sin^2\theta \cos^2\theta \quad . \quad . \quad . \quad (14)$$

Since $PM = a = c \sec \beta$, and $OM = c \tan \beta$ we have from the triangle POM

$$c^2 \sec^2 \beta = r^2 + c^2 \tan^2 \beta - 2rc \tan \beta \sin \theta$$

i.e.,

$$(r^2 - c^2)/2rc = \tan \beta \sin \theta$$

But

$$y = \sin \theta (r^2 - c^2)/r$$
$$= 2c \tan \beta \sin^2 \theta$$

Substituting for θ in (14) we obtain

$$x^2 + \{ y + 2c \cot 2\beta \}^2 = (2c \operatorname{cosec} 2\beta)^2$$

showing that the locus of P′ is an arc of a circle of radius $2c$ cosec 2β and centre the point $(0, -2c \cot 2\beta)$. The point B will transform into the point B', $z = -2c$, and the point A into the point A', $z = +2c$.

It is easily seen that the tangent to the arc at B' makes an angle 2β with the x axis.

The chord $B'A' = 4c$, while if the arc $B'A'$ cuts the η axis in T, then $0T = 2OM$, giving

$$\text{camber} = 2OM/4c = \tfrac{1}{2}c \tan \beta.$$

From the fact that equation (14) remains unchanged when $-\theta$ is substituted for θ it follows that as P describes the circle, P' describes the arc $B'TA'$ twice. The above results are illustrated geometrically in Appendix II.

5 *Joukowsky Aerofoils*

We return to the case when M does not lie on the η axis. Let $MM_0 = mc$ (Fig. 5) and assume m and β small. Then if z be a point P' on the aerofoil corresponding to a point P, $\zeta = re^{i\theta}$ on the circle we can find approximate expressions for the co-ordinates of P'.

Then to the first order $a = c(1 + m)$ and the co-ordinates of centre M are $\xi = mc$, $\eta = \beta c$.

Since P is on the circle centre M and radius a we have

$$(r \sin \theta - mc \sin \beta - c \tan \beta)^2 + (r \cos \theta - mc \cos \beta)^2$$
$$= (c \sec \beta + mc)^2$$

Retaining only terms of the first order we have

$$r^2 - 2rc(\beta \sin \theta + m \cos \theta) - c^2(1 + 2m) = 0$$

Then

$$r/c = 1 + m(1 + \cos \theta) + \beta \sin \theta$$
$$c/r = 1 - m(1 + \cos \theta) - \beta \sin \theta$$

Now

$$z = \zeta + c^2/\zeta$$
$$x = (r + c^2/r) \cos \theta$$
$$y = (r - c^2/r) \sin \theta$$

whence

$$x/c = 2 \cos \theta$$
$$y/c = 2 \sin \theta \{ m(1 + \cos \theta) + \beta \sin \theta \}$$

When $\theta = 0$, $x = 2c$, and when $\theta = \pi$, $x = -2c$, hence the chord $A'B'$ (Fig. 5) of the aerofoil is $4c$ approximately.

If P and P_1 are points on the circle such that OP and OP_1 make angles θ and $-\theta$ with $O\xi$ and y and y_1 are the ordinates of the corresponding points on the aerofoil we have, if t is the thickness of the aerofoil

$$t = y - y_1$$
$$= 4mc \sin \theta (1 + \cos \theta)$$

This will be a maximum when $dt/d\theta = 0$, giving $\cos\theta = \frac{1}{2}$, *i.e.*, thickness is a maximum when $x = c$, *i.e.*, at a distance from leading edge equal to about $\frac{1}{4}$ chord. At this point $t = 3\sqrt{3}mc$.

Again

$$\tfrac{1}{2}(y + y_1) = 2c\beta \sin^2\theta$$

which is a maximum when $\theta = \pi/2$ and hence mean camber $= 2\beta c/4c = \beta/2$.

Again, since the circle centre M would touch a circle centre M_0 and radius BM_0 (the dotted circle in Fig. 5) at B, it follows that the aerofoil and circular arc into which these two circles are transformed by means of (13) will have the same tangent at B'. Hence the Joukowsky aerofoil has a cusp at the trailing edge, the cuspidal tangent making an angle 2β with the chord.

In Appendix II. a method of constructing a Joukowsky aerofoil is given and some geometrical properties of the aerofoil are proved. Also in Fig. 16 the flow round the aerofoil is illustrated.

Finally

$$k_L = L/b\rho V^2 = (4a/b)\,\pi \sin(\alpha + \beta)$$

where b is the chord of the aerofoil.

Since $a = (1 + m)\,c$, and $b = 4c$ approximately for Joukowsky aerofoils

$$k_L = (1 + m)\,\pi \sin(\alpha + \beta)$$

For wing sections m lies between 0.1 and 0.05, hence to a first approximation

$$dk_L/d\alpha = \pi$$

i.e., the slope of the lift curve is π per radian, or 0.055 per degree for aerofoils of infinite aspect ratio. This value corresponds to a slope of 0.041 per degree for aerofoils of aspect ratio 6.

The slopes obtained from wind channel tests on models of some typical wing sections of aspect ratio 6 are given below. The average slope is 0.041, thus giving good agreement with the above theory.

R. & M.	WING.	SLOPE.	R. & M.	WING.	SLOPE.
323	R.A.F. 14	0.0413	377	Bristol F2B	0.0412
774	R.A.F. 15	0.0391	375	Albatross	0.0408
415	R.A.F. 18	0.0406	481	Curtiss	0.0425

Strut Shape.—In the case when M lies on the ξ axis $\beta = \gamma = 0$ and the first and second axes coincide. Then the circle transforms into a symmetrical strut-shaped section (Fig. 7) having a fixed centre of pressure, *i.e.*, the lines of action of the lift all pass through a point F in BM such that $MF = c^2/a$.

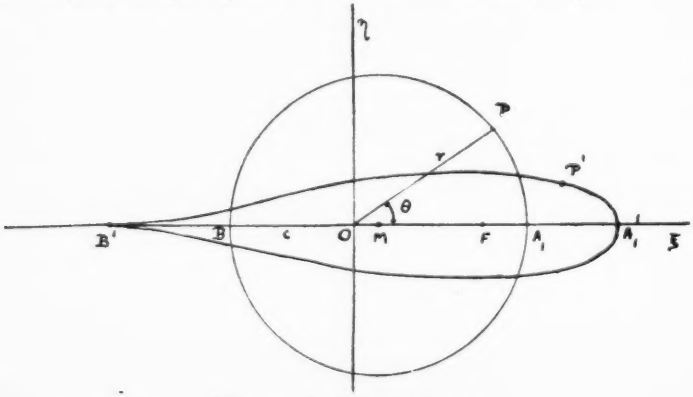


FIG. 7.—*Joukowsky Strut.*

The co-ordinates of a point P' on the strut corresponding to a point P, $\zeta = re^{i\theta}$ on the circle are now given by

$$x/c = 2\cos\theta,\ y/c = 2m\sin\theta\,(1 + \cos\theta)$$

The maximum thickness is $3\sqrt{3}mc$ and occurs when $x = c$.

6 *Summary of Results for Joukowsky Aerofoils*

Given a circle

$$|\zeta - \zeta_0| = a$$

in the ζ plane where the radius a and the centre M, $\zeta = \zeta_0$, are determined by means of the quantities $BO = c$, $MBO = \beta$, $MM_0 = cm$ (Fig. 5), then by means of the transformation

$$z = \zeta + c^2/\zeta$$

the circle is transformed into an aerofoil having a cusp at the trailing edge and a chord of length $4c$ approximately. The cuspidal tangent makes an angle 2β with the chord.

The lift is given by

$$L = 4\pi a\rho V^2 \sin(\alpha + \beta)$$

and the moment of the lift about the centre M by

$$M_1 = 2\pi c^2\rho V^2 \sin 2\alpha$$

The slope of the lift curve is approximately π per radian.

If F is the point such that vectorially $MF = (c^2/a)\,e^{-i\beta}$ the lines of action of the lift all touch a parabola whose focus is F and directrix BM.

The moment of the lift about F is given by

$$M = -2\pi c^2\rho V^2 \sin 2\beta$$

and is seen to be independent of α. Further, the perpendicular h from F on the line of action of the lift is given by

$$h = (c^2 \sin 2\beta)/2a \sin(\alpha + \beta)$$

The mean camber is $\beta/2$ approximately, hence when $\beta = 0$, we obtain a strut-shaped section, and since in this case $h = 0$, the strut will have a fixed centre of pressure.

The maximum thickness is $3\sqrt{3}\,mc$ approximately. Then as $m \to 0$, the thickness$\to 0$ and in the limit we have a circular arc of chord $4c$ and height $2c \tan\beta$.

To a first approximation $a = (1 + m)\,c$. Thus from the form of h it appears that the movement of the centre of pressure increases with increase of camber and decreases with increase of thickness.

7 *The Transformation* $(z - nc)/(z + nc) = (\zeta - c)^n/(\zeta + c)^n$.

The transformation $z = \zeta + c^2/\zeta$ may be written

$$(z - 2c)/(z + 2c) = (\zeta - c)^2/(\zeta + c)^2 \quad . \quad . \quad . \quad (15)$$

This is a more particular case of the more general transformation

$$(z - nc)/(z + nc) = (\zeta - c)^n/(\zeta + c)^n \quad . \quad . \quad . \quad (16)$$

Then the points $\pm c$ in the ζ plane transform into the points $\pm nc$ in the z plane.

We may write for equation (16)

$$z - nc = (\zeta - c)^n \phi_1(\zeta) \quad . \quad . \quad . \quad . \quad (16a)$$

where $\phi_1(\zeta)$ and $d\phi_1/d\zeta$ are finite, continuous and different from zero in the neighbourhood of $\zeta = c$.

Similarly we may write equation (16) in the form

$$z + nc = (\zeta + c)^n \phi_2(\zeta) \quad . \quad . \quad . \quad . \quad (16b)$$

where $\phi_2(\zeta)$ and $d\phi_2/d\zeta$ are finite, continuous and different from zero in the neighbourhood of $\zeta = -c$. Then it follows that $dz/d\zeta$ has zeros of order $(n-1)$ at the points $\zeta = \pm c$.

Returning to equation (16)

$$(z - nc)/(z + nc) = (1 - c/\zeta)^n/(1 + c/\zeta)^n$$

Expanding in descending powers of $1/\zeta$ we have

$$z = \zeta + \{(n^2 - 1)/3\}(c^2/\zeta) + \ldots \quad . \quad . \quad . \quad (16c)$$

Thus in this case $c_1 = (n^2 - 1)c^2/3$, and $\gamma = 0$.

The transformation (16) is applied as before to a circle with centre M and radius a, the point $\zeta = -c$ being a point B on the circumference and the point $\zeta = c$ lying within the circle (Fig. 8). $MBO = \beta$, and BM cuts the

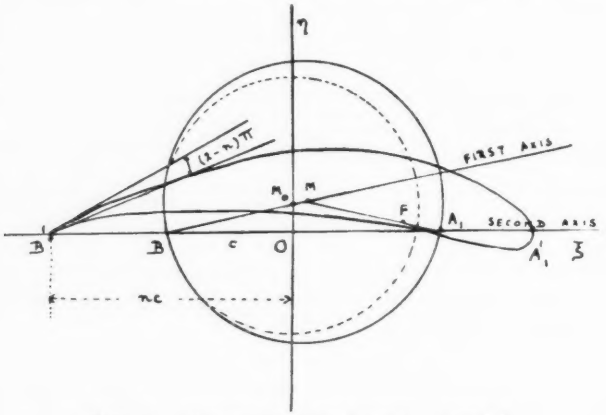


FIG. 8.—*Generalised Joukowsky Aerofoil.*

η axis in M_0. Then BM is the first axis, BO the second axis and the point B transforms into the point B', $z = -nc$. Applying the general theory we have

$$L = 4\pi a \rho V^2 \sin(\alpha + \beta)$$

The moment of L about centre $M = \frac{2}{3}c^2(n^2 - 1)\rho V^2 \sin 2\alpha$. The focus F is a point in MA_1 such that $MF = (n^2 - 1)c^2/3a$. The moment of L about $F = -\frac{2}{3}\pi c^2(n^2 - 1)V^2 \sin 2\beta$.

Again the most suitable types of aerofoils are obtained by taking MM_0 and β small.

As before we first consider the limiting case when $MM_0 = 0$, although the contours so obtained will not belong to the class of aerofoils under consideration as now there are zeros of $dz/d\zeta$ at two points on the circumference of the circle, *i.e.*, at $\zeta = \pm c$. The equations of motion would then give infinite velocity at the point on the contour corresponding to the point $\zeta = +c$ on the circle.

8 *The Double Circular Arc*

When M lies on the η axis the circle passes through the points $\zeta = \pm c$ and these points transform into the points $z = \pm nc$ (Fig. 9).

The transformation (16) may be written vectorially as

$$P'A'/P'B' = (PA/PB)^n \quad . \quad . \quad . \quad . \quad (17)$$

Denoting PA and PB by r_1 and r_2 and the angles they make with the ξ axis by θ_1 and θ_2 and adopting a similar notation with dashed letters for corresponding lines and angles in the z plane, we may write for equation (17)

$$r'_1 e^{i\theta'_1} / r'_2 e^{i\theta'_2} = \{ r_1 e^{i\theta_1} / r_2 e^{i\theta_2} \}^n$$

i.e.,

$$(r'_1/r'_2)\, e^{i(\theta'_1 - \theta'_2)} = (r_1/r_2)^n\, e^{in(\theta_1 - \theta_2)}$$

whence

$$\theta'_1 - \theta'_2 = n(\theta_1 - \theta_2)$$

i.e.,

$$A'P'B' = nAPB = n\psi \text{ say.}$$

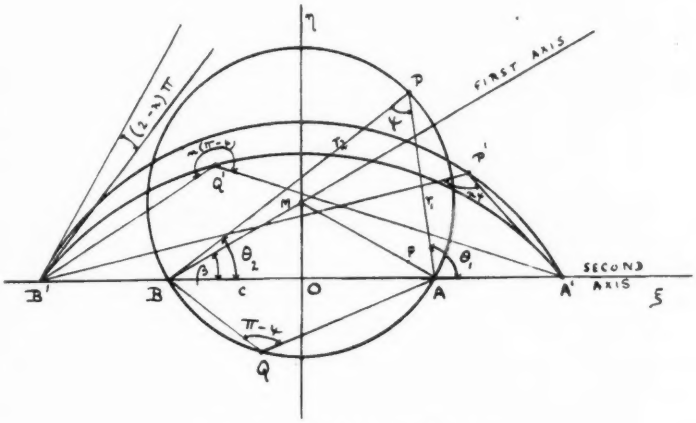


FIG. 9.—*Double Circular Arc.*

Now as P moves round the major arc APB of the circle ψ is constant, therefore as P' moves from A' to B' the angle $A'P'B'$ remains constant, *i.e.*, P' describes a circular arc.

Again, if Q is a point on the minor arc of the circle and Q' the corresponding point in the z plane, then reflex angle

$$A'Q'B' = nAQB = n(\pi - \psi)$$

$$\therefore\ A'Q'B' = 2\pi - (n\pi - \psi) = (2 - n)\pi + n\psi$$

But as Q describes the arc BQA, angle BQA remains constant and therefore as Q' moves from B' to A' angle $B'Q'A'$ remains constant, *i.e.*, Q' describes another circular arc.

Now it can easily be shown that two arcs standing on a common chord subtending angles α and β at the circumferences cut at an angle $\beta - \alpha$. Hence the arcs $A'P'B'$, $A'Q'B'$ cut at an angle τ where

$$\tau = (2 - n)\pi + n\psi - n\psi = (2 - n)\pi$$

For a small angle at the trailing edge we must take n slightly less than 2.

The circle will transform into a sickel providing

$$nAQB > \pi$$

i.e.,

$$n(\pi - \psi) > \pi$$

But $\psi = \pi/2 - \beta$, therefore we must have $\beta > \{ 1/n - 1/2 \}\pi$.

9 *The Generalised Joukowsky Aerofoil*

To return to the case when the point $\zeta = c$ lies within the circle. Then the aerofoils obtained by means of the transformation (16) will conform to the general theory and the flow round them will be streamline.

Since in practice n is taken slightly less than 2 the new series of aerofoils are of the same type as the Joukowsky aerofoils, but with a small angle at the trailing edge. In Fig. 8 n has been taken equal to 1.95, giving an angle of 9° at the trailing edge.

Since (Fig. 8) the circle with centre M touches the circle with centre M_0 and radius BM_0 at B, it follows that the aerofoil and double circular arc into which these circles are transformed by means of (16) will have the same tangents at B', hence the aerofoil has an angle $(2 - n)\pi$ at the trailing edge.

It can be shown that, assuming MM_0 and β to be small, the length of the chord $A'B'$ is approximately $4nc$.

Strut Shape.—In the case when M lies on the ξ axis, $\beta = \gamma = 0$, and we obtain a strut-shaped section (Fig. 10), having an angle $(2 - n)\pi$ at the trailing edge.

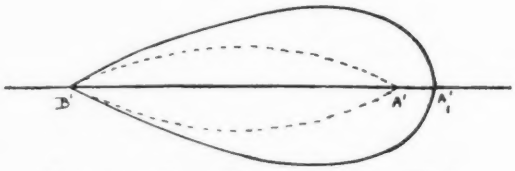


FIG. 10.—*Generalised Joukowsky Strut.*

Since $\beta = \gamma = 0$, the first and second axes are parallel and the strut has a fixed centre of pressure.

It may be noted that in both the Joukowsky type of aerofoils, and in the generalised type, the symmetrical case is the only one which gives a section having a fixed centre of pressure. As it would obviously be of interest to obtain non-symmetrical shapes with a fixed centre of pressure, transformations have been sought for which would produce the required result.

Mises by using three terms of the transformation (2), and by a suitable choice of the point B and the coefficients a_1 and a_2 obtains an aerofoil section (Fig. 11) having a fixed centre of pressure. For more detailed information about this aspect of the subject which provides considerable scope for further investigation the reader is referred to Professor Mises' article in the "Zeitschrift für Flugtechnik und Motorluftschiffahrt," Vol. II. (1920), page 86.

APPENDIX I.

Proof of the Moment Formula

1. *Complex Quantities.*

The complex quantity $z = x + iy$, where $i = \sqrt{-1}$ may be represented in the z plane by the point P whose co-ordinates referred to rectangular axes are (x, y).

If r, θ are the polar co-ordinates of P we have

$$\begin{aligned} z &= x + iy \\ &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

r is always positive and is called the modulus of the complex quantity z and is denoted by $|z|$. θ is called the argument of z and is denoted by arg z.

2. *Stream and Potential Functions.*

If we consider the irrotational motion of an inviscid fluid in two dimensions it can be shown that a potential function $\phi(x, y)$ and a stream function $\psi(x, y)$ exist such that the lines $\phi(x, y) =$ constant give the lines of equi-velocity-potential, and the lines $\psi(x, y) =$ constant give the stream lines.

The above is equivalent to the statement that we can find $\phi(x, y)$ and $\psi(x, y)$ such that

$$w = \phi + i\psi = f(x + iy) = f(z)$$

Then ϕ and ψ satisfy the relations

$$\delta^2\phi/\delta x^2 + \delta^2\phi/\delta y^2 = 0, \quad \delta^2\psi/\delta x^2 + \delta^2\psi/\delta y^2 = 0,$$

and if u, v are components of the velocity q at any point in the plane parallel to the co-ordinate axes

$$u = -\delta\phi/\delta x = -\delta\psi/\delta y$$
$$v = -\delta\phi/\delta y = \delta\psi/\delta x$$

The form of ϕ and ψ will depend on the conditions at the boundary of the fluid.

Taking

$$w = \phi + i\psi$$
$$dw/dz = \delta w/\delta x = \delta\phi/\delta x + i\delta\psi/\delta x$$
$$= -u + iv$$
$$= -q(\cos\delta - i\sin\delta)$$
$$= -qe^{-i\delta}$$

where δ is the angle made by the resultant velocity with the x axis.

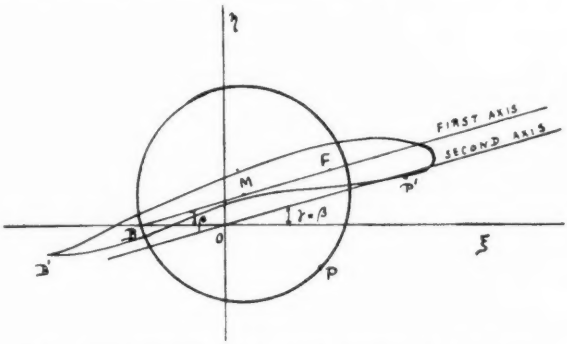


FIG. 11.—*Aerofoil with Fixed Centre of Pressure.*

Moment Formula

Referring to Fig. 2 and equation (4) of the present paper, we find that the motion in the ζ_1 plane is given by

$$w = \phi + i\psi = + V(\zeta_1 + a^2/\zeta_1) + (iK/2\pi)\log\zeta_1$$

If now we transfer the origin to the point o (Fig. 1) and take axes through this point inclined at an angle α to the axes in the ζ_1 plane, we have, taking $\zeta = \zeta_0$ to be the point M in the ζ plane,

$$\zeta_1 = (\zeta - \zeta_0)e^{i\alpha}$$
$$\therefore w = V\{(\zeta - \zeta_0)e^{i\alpha} + a^2e^{-i\alpha}/(\zeta - \zeta_0)\} + (iK/2\pi)\{\log(\zeta - \zeta_0) + i\alpha\}$$
$$\therefore dw/d\zeta = V\{e^{i\alpha} - a^2e^{-i\alpha}/(\zeta - \zeta_0)^2\} + (iK/2\pi)/(\zeta - \zeta_0)$$
$$= Ve^{i\alpha} + (iK/2\pi)/\zeta + (-Va^2e^{-i\alpha} + iK\zeta_0/2\pi)/\zeta^2 + \ldots$$

Now

$$z = \zeta + a_1/\zeta + a_2/\zeta^2 + \ldots$$

$$dz/d\zeta = 1 - a_1/\zeta^2 - 2a_2/\zeta^3 - \ldots$$

$$\therefore\ dw/dz = (dw/d\zeta)/(dz/d\zeta)$$

$$= (dw/d\zeta) \times (1 + a_1/\zeta^2 + \ldots)$$

$$= Ve^{i\alpha} + (iK/2\pi)/\zeta + (Va_1e^{i\alpha} - Va^2e^{-i\alpha} + iK\zeta_0/2\pi)/\zeta^2 + \ldots$$

$$\therefore\ (dw/dz)^2 = V^2e^{2i\alpha} + (iKVe^{i\alpha}/\pi)/\zeta$$

$$+ (-K^2/4\pi^2 + 2V^2a_1e^{2i\alpha} - 2V^2a^2 + iKV\zeta_0e^{i\alpha}/\pi)/\zeta^2 + \ldots \quad \text{(i.)}$$

Now in the z plane, *i.e.*, the plane of the aerofoil, consider a region of fluid bounded by the aerofoil and by a large circle with centre o and radius R (Fig. 12).

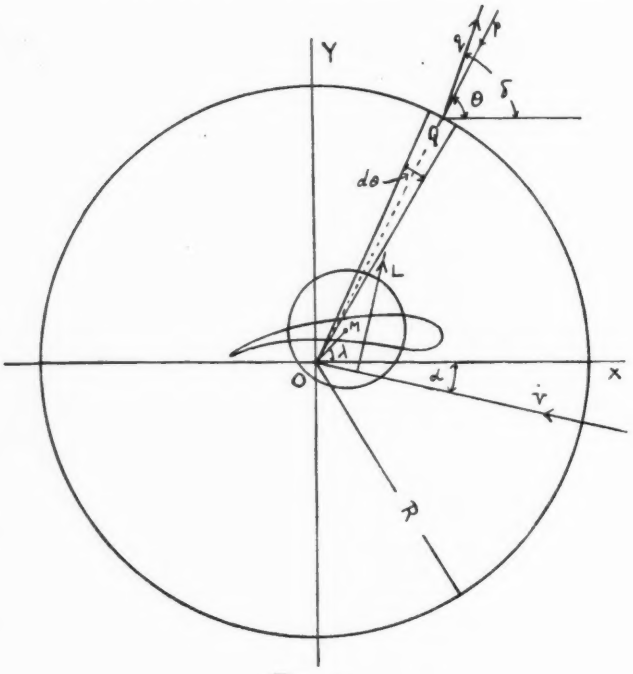


FIG. 12.

Then the excess of the momentum of the fluid leaving this region over that entering it in unit time is equal to the sum of the external forces, namely, the reaction $-L$ of the aerofoil and the pressures on the boundary of the circle. Again the moment of this excess of momentum about any point is equal to the sum of the moments of the external forces about that point. Since the pressure on the boundary of the circle is everywhere normal to it, the moment of the pressure about o vanishes.

Let ds be an element of arc at a point Q on the boundary of the circle and let OQ make an angle θ with Ox. At Q the velocity q makes an angle δ with ox.

Then $\rho q \cos(\delta - \theta)\, ds$ represents the quantity of fluid flowing across the boundary ds in unit time, ρ being the density of the fluid. Since the velocity

perpendicular to OQ is $q \sin(\delta - \theta)$ the moment of momentum of this fluid about o is $\rho R q^2 \cos(\delta - \theta) \sin(\delta - \theta)\, ds$.

Equating the integral of this quantity taken round the circle to the moment of the external forces about o we have, if M is the moment of the lift L about o,

$$-M = \rho \int_0^{2\pi} R^2 q^2 \cos(\delta - \theta) \sin(\delta - \theta)\, d\theta$$

$$= \tfrac{1}{2}\rho R^2 \int_0^{2\pi} (dw/dz)^2\, e^{2i\delta} \sin 2(\delta - \theta)\, d\theta$$

Now $-\sin 2(\delta - \theta)$ = imaginary part of $e^{-2i(\delta - \theta)}$

$$= Ie^{-2i(\delta - \theta)} \text{ say.}$$

$$\therefore M = \tfrac{1}{2}\rho R^2 \{ I \int_0^{2\pi} (dw/dz)^2\, e^{2i\theta}\, d\theta \}$$

Now substituting for $(dw/dz)^2$ the expansion in terms of $1/\zeta$ obtained from (i.), and putting $\zeta = Re^{i\theta}$ we have a series of terms of the type

$$\int_0^{2\pi} e^{in\theta}\, d\theta$$

where $n = 2, 1, 0, -1 \ldots$

But

$$\int_0^{2\pi} e^{in\theta}\, d\theta = 0 \text{ if } n \text{ does not} = 0$$

$$= 2\pi \text{ if } n = 0$$

Hence

$$M = \tfrac{1}{2}\rho I (2V^2 a_1 e^{2i\alpha} + iKV\zeta_0 e^{i\alpha}/\pi)\, 2\pi$$

$$= \rho I (2\pi V^2 c_1^2 e^{2i(\alpha + \gamma)} + iKVr_0 e^{i(\alpha + \lambda)})$$

where

$$a_1 = c_1^2 e^{2i\gamma} \text{ and } \zeta_0 = r_0 e^{i\lambda}$$

$$\therefore M = 2\pi c_1^2 \rho V^2 \sin 2(\alpha + \gamma) + K\rho V r_0 \cos(\alpha + \lambda)$$

whence if M_1 be the moment of L about the centre M

$$M_1 = 2\pi c_1^2 \rho V^2 \sin 2(\alpha + \gamma)$$

The above proof is given in an article by Mises,* who has also given a proof of the formula $L = \rho VK$ along the same lines.

APPENDIX II.

I *The Circular Arc Treated Geometrically*

In paragraph 4 of the present paper it was shown that when the centre M of the circle of transformation (Fig. 6) lay on the η axis, the circle transformed into a circular arc.

* "Zeitschrift für Flugtechnik und Motorluftschiffahrt," Vol. 8 (1917), page 161.

P is the point $\zeta = re^{i\theta}$ on the circle.

P_1 is the point $\zeta = r_1 e^{-i\theta}$.

Now if PO is produced to meet the circle again in P_2 we have

$$OP \times OP_2 = c^2$$
$$\therefore\ OP_1 = OP_2 = c^2/r$$

hence P_1 is the point $(c^2/r)\, e^{-i\theta}$.

But the point P' on the arc corresponding to the point P on the circle is given by

$$z = \zeta + c^2/\zeta$$
$$= re^{i\theta} + (c^2/r)\, e^{-i\theta}$$

i.e., is found by adding the vectors OP and OP_1. Hence to obtain the point P' we complete the parallelogram POP_1P'.

It follows from the above that P and P_1 each transform into the point P' and hence P' describes the arc $B'TA'$ twice as P describes the circle.

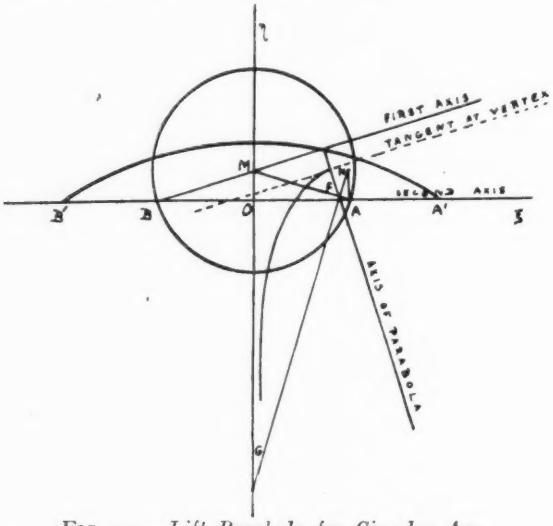


FIG. 13.—*Lift Parabola for Circular Arc.*

The focus F is a point in MA (Fig. 13) such that $MF = c^2/a$. The point is found geometrically by erecting a perpendicular AN to AB at A of length equal to OM, and drawing NF perpendicular to MA. It will be found that NF produced passes through the centre G of the arc $B'A'$.

The axis of the lift parabola is the line through F perpendicular to BM, the directrix.

2 *Construction of a Joukowsky Aerofoil*

We obtain a Joukowsky aerofoil when the centre M of the circle of transformation is taken so that the line joining B and M cuts the η axis (Fig. 14).

Let P be the point $\zeta = re^{i\theta}$ on the circle K with centre M and radius BM.

Then the corresponding point P' on the aerofoil is given by

$$z = \zeta + c^2/\zeta$$
$$= \zeta + \zeta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where

$$\zeta_1 = (c^2/r)\, e^{-i\theta} = r_1 e^{-i\theta}$$

Now the point P_1 given by $\zeta = \zeta_1$ is such that OP_1 is the inverse of OP with respect to a circle centre o and radius c, while OP_1 makes an angle $-\theta$ with the ξ axis.

But it is well known that the inverse of a circle is a second circle. Hence as P moves round the circle K, P_1 will move round a second circle K_1. Moreover, a circle K_0 with centre M_0 and radius M_0B will touch circle K at B. Again the circle K_0 inverted with respect to a circle centre o and radius c and then reflected traces out the same circle. Now since on inversion the angle at which two curves cut remains unchanged, it follows that circle K_0 will touch circle K_1 at B. Therefore the centre M_1 of circle K_1 lies on BM. But the circle K when inverted becomes a circle with its centre in MO produced. Hence, when this latter circle is reflected about the ξ axis its centre M_1 will lie on a line such that the angle M_1OM_0 = the angle M_0OM. Thus the position of M_1 in the line BM is determined, and since the circle K_1 passes through B it can now be drawn.

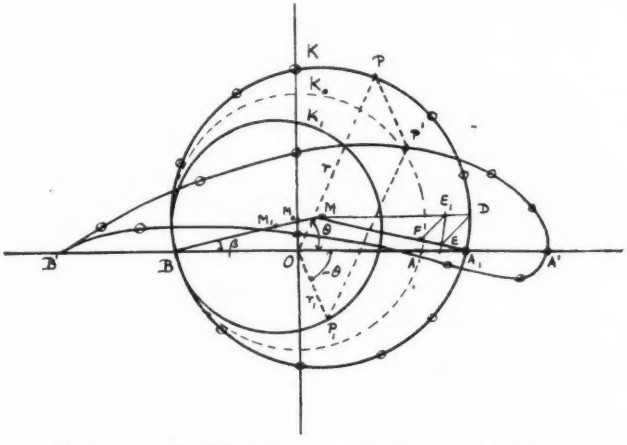


FIG. 14.—*Construction of a Joukowsky Aerofoil.*

The point P_1 on this circle K_1 is such that OP_1 makes an angle $-\theta$ with the ξ axis.

Then from equation (18) we see that the point P' is found by compounding the vectors OP and OP_1 by the parallelogram law.

Thus, by taking positions of P for which $\theta = 10°$, $20°$, $30°$, etc., a series of corresponding points on the aerofoil can be found.

As $\gamma = 0$, the focus F is a point in the line MA_1 such that $MF = c^2/a$.

To find F by a geometrical construction draw through M a line parallel to the ξ axis to meet circle K in D. With centre M and radius c draw an arc to cut MA_1 in E and MD in E_1. Draw E_1F parallel to DE to meet MA_1 in F.

Then F is the focus of the lift parabola and BM the directrix. The line of action of the lift for any given angle of incidence is then easily determined (Fig. 4).

In a previous paper* I have obtained the streamlines round the aerofoil from those round the given circle by a method similar to that used in constructing the

* R. & M., 788. Stream-lines round a Joukowsky Aerofoil.

aerofoil. The flow past the circle and the corresponding flow past the aerofoil are illustrated in Figs. 15 and 16.

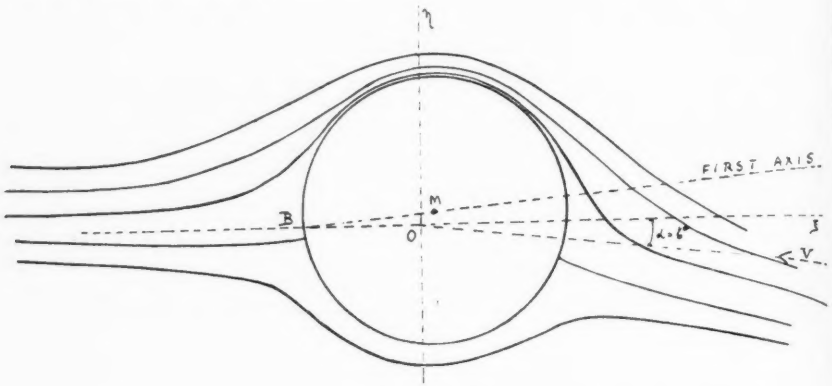


FIG. 15.—*Streamlines Round a Circular Cylinder with Circulation.*

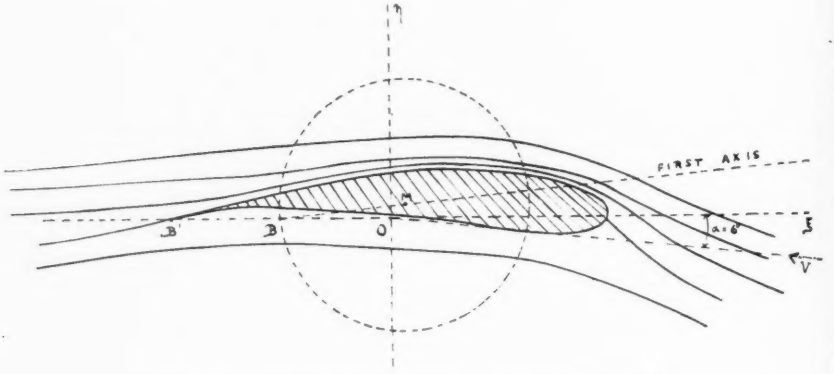


FIG. 16.—*Streamlines Round a Joukowsky Aerofoil.*

REVIEWS

Sur la Theorie des Surfaces Portantes

M. Roy. Gauthier-Villars et Cie, 1922.

Prandtl's aerofoil theory is receiving considerable attention at the present time, and M. Roy has now given a short account of the basis and development of the theory. Starting with a short summary of the fundamental equations of hydrodynamics, the author passes on to discuss the idea of circulation and the method in which it arises. After this point the lines of Prandtl's original papers are followed very closely, and the problems of the monoplane, biplane and wind channel interference are discussed in turn. The author concludes with a short account of the minimum energy losses of an airscrew as developed by Betz.

The book contains little that is new to the student of modern aerofoil theory, but it undoubtedly gives a good account of Prandtl's work, which will be of value to those who cannot obtain access to Prandtl's original papers. The author has also provided an interesting introduction to the fundamental ideas of the theory, but the student must be warned that the proof of the relationship connecting lift and circulation given on page 27 is quite unsound.

The Evolution of Climate

By C. E. P. Brooks, M.S.C., F.R.A.I., F.R.Met.Soc. With a Preface by G. C. Simpson, D.Sc., F.D., Director of the Meteorological Office.

In this book Mr. Brooks discusses the various theories to account for the past glaciation of the earth, and then gives his own conception of the cause. He gives also a more or less precise geological history of the various countries in which proofs of past glaciation have been found. He comments on the vast amount of literature on the subject and on the impossibility of any one man assimilating it all.

Mr. Brooks comes to the conclusion that the various hypotheses put forward to account for the past changes of climate are untenable. However, he fails to state the most conclusive argument against Croll's theory, namely, that whatever the combination of the eccentricity of the earth's orbit and the position of the apse may be, the total heat received in the year on each hemisphere remains invariable. If the total heat received is unaltered, it is difficult to believe that the mean annual temperature can be greatly altered by its different distribution. He also turns down the theory put forward by Tyndall and Arrhenius that the ice ages were due to changes in the composition of the atmosphere, probably correctly. But the question of the effect of radiation on atmospheric temperature is so complicated and difficult that it is hardly safe to dogmatise upon it, and it seems possible, though improbable, that changes in the amount of carbonic acid in the atmosphere might have some effect upon the temperature.

Mr. Brooks' own suggestion is that the changes in the distribution of land and sea may produce large changes of temperature. By tabulating the temperatures of various regions of the earth and correlating them with what he calls the "continentality" of the district, he has found a close agreement between the quantities. His conclusions are that in winter the effect of land to the West in temperate latitudes is always to lower temperature; that the effect of land to the East is almost negligible. In summer the general effect of land, whether to East or West, is to raise temperature, but the effect is not nearly so marked as the opposite effect in winter (see Quarterly Journal of R. Met. Soc., Vol. XLIII., 1917, p. 169, and Vol. XLIV., p. 253).

Mr. Brooks' conclusions are most interesting, and are well worth careful study. He considers also that a further cause of change of climate will be produced by a change of distribution of land and sea, because the anticyclonic areas would take up a different position and the usual track of cyclonic disturbances would change.

In our present state of knowledge, or want of knowledge, as to the cause of

cyclones and anticyclones, we may perhaps safely say that this would happen, but it is very difficult to give any detail. Would, for example, any three meteorologists agree as to the track that cyclones would follow if Scandinavia were glaciated? It must also be remembered that glaciation depends on snow-fall as well as on temperature. Raising the land to a sufficient height would produce glaciation, because it would lower the temperature and also cause copious snowfall. But large parts of Canada and Siberia have mean annual temperatures far below the freezing point, as is evidenced by the permanently frozen ground, and yet no glaciation. Land to the westward in temperate latitudes cuts off winter rain, as well as lowers the temperature.

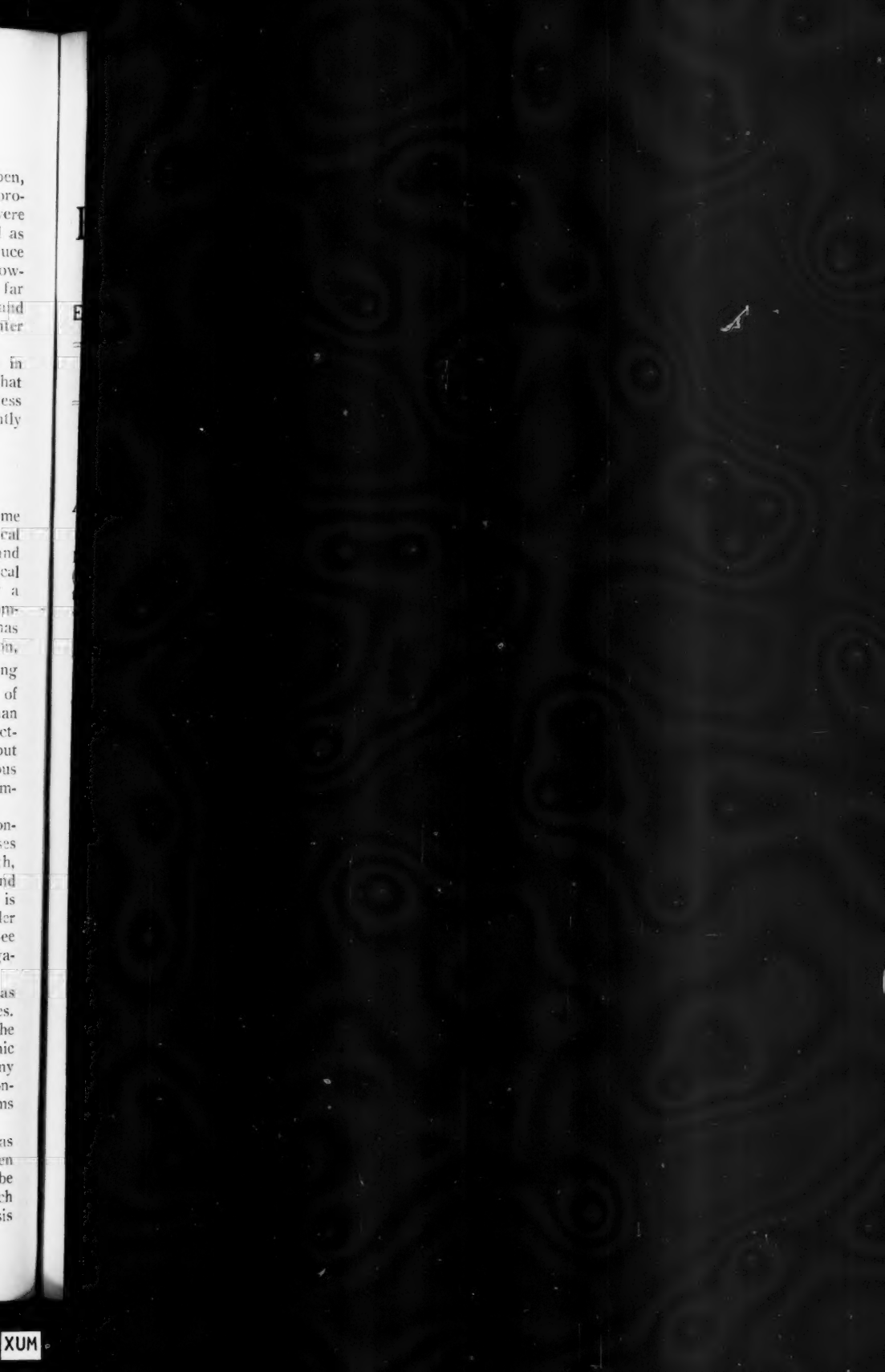
The chief part of the book gives some account of the various ice-ages in different countries. Perhaps the statements are given without sufficient hint that they are not generally recognised facts, but are facts mixed with more or less conjecture. But the book is very interesting and suggestive, and can be confidently recommended.

A Treatise on Engine Balance Using Exponentials

By P. Cormac.

Mr. P. Cormac's new book on Engine Balance, which forms a welcome addition to the literature of the subject, will be of special interest to aeronautical engineers in that it treats somewhat fully of aero engines, including radial and rotary engines. It is essentially a book for a student of considerable mathematical ability. Instead of the usual elementary graphical methods of balancing a number of rotating forces, or the alternative of expressing the resolved components in two directions at right angles by the circular functions, the author has obtained his results by the use of the exponential method of vector representation, in which a rotating force is denoted by $Z_0 e^{i\theta}$ and a force in one direction varying harmonically $Z_0 \cos\theta$ becomes $\frac{1}{2} Z_0 (e^{i\theta} + e^{-i\theta})$. The addition of a number of forces from a corresponding number of cylinders is much easier to evaluate than a series of circular functions, and the results are thus obtained with great directness. The proofs are of the simplest to any student of higher mathematics, but it is probable that the great majority of engineers will prefer the more cumbrous methods. The author has, however, provided a useful first chapter on the complex function, which should be sufficient for those unfamiliar with its use.

As already mentioned, in addition to the multi-cylinder-in-line engine, considerable space is devoted to radial and rotary engines, and it is in these cases that the greatest simplification occurs. The effect of off-setting is dealt with, and also certain special forms of engine such as the Leitlin rotary engine and oscillating cylinder engines, but it is remarkable that the book throughout is confined to engines in which the motion and forces in connection with each cylinder are identical. In view of the preponderance of makes of radial, rotary and Vee engines with master connecting rod assembly, it is regretted that some investigation into the considerable differences thus introduced was not attempted.

In addition to the study of the balance of the whole engine, the author has applied his method to an examination of the torque curve of multi-cylinder engines. As it is first necessary to obtain the torque curve for a pair of cylinders from the indicator diagram, and then to analyse it into the sum of a series of harmonic functions, it is difficult to see that the resultant torque will be obtained with any greater ease or accuracy than by the method of arithmetical addition. The concluding chapters are devoted to the reverse problem of the design of crank systems with specified balance.

Although it is considered that the book could be considerably improved, as indicated by a fuller treatment of the more common types of aero engines, even at the expense of other less usual types, yet it contains much which is not to be found in present text books on the subject, and indicates a method of approach to many similar problems often left unsolved because of the cumbrous analysis involved.